Toward a Framework for Navigating Simulation Fidelity in Model-Based Design

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Abstract

Model-Based Design is a sequential decision process that increases the detail of modeling and analysis while simultaneously decreasing the space of alternatives considered. Computational models refine sequentially from conceptual models that are used for initial trades and progress through models of increasing detail until the final technical data package of the design is subject to the highest fidelity analysis. Our premise is supported by research from the domains of behavioral economics, psychology, judgment and decision making, neuroeconomics, marketing, engineering design, and our previous experience performing trade studies with numerous companies and government agencies. This paper presents a preliminary framework whereby the design process itself can be modeled as a series of discrete steps through a three-dimensional space consisting of model detail, analysis complexity, and design set size, with time, cost, and the quality of information derived attached to each step. With this framework, we are able to now consider fundamental questions such as what is the best rate of increase in detail, decrease in attribute sets, and how many discrete modeling steps should be taken? Issues with the application of this framework for design as a sequential process are highlighted.

Keywords
simulation, fidelity, model-based design, trade space

1. Introduction and Motivation

The paper first introduces the inherent properties of design as a sequential process, followed by asking questions concerning navigating the process, and finally a preliminary model of design under a proposed framework. This framework casts the navigation of a trade space into a problem of traversing a three-dimensional space defined by model detail, analysis complexity, and design set size. The paper concludes with questions about the application of the framework, presents ongoing research in this area, and provides suggestions for future efforts.

2. Background

The premise that design decision making should be treated as a sequential decision process rather than as a static decision problem is bolstered by research from the domains of behavioral economics [1], judgment and decision making [2], neuroeconomics [3], marketing [4], and engineering design [5, 6]. It is further substantiated by the authors’ own experience in conducting trade studies for numerous customers across engineering domains [6]. Miller et al. [7] provide a comprehensive review of the relevant literature.

Design process models take many forms from Ulrich and Eppinger’s Product Design and Development [8] to Finger and Dixon’s [9] argument for a descriptive process. The latter states that process models are better presented as a “…descriptive model that explains how design is done, a cognitive model that explains the designer’s behavior, a prescriptive model that shows how design must be done, and a computable model that expresses a method by which a
computer can accomplish a task” [9]. These process models all present challenges for programmatic implementation as design requirements are an evolving construct. This lends treating design as a sequence of refinements in terms of modeling fidelity, accuracy, and representation appropriate.

Abstraction and granularity are synonyms for describing the level of modeling detail and complexity in design, analysis, and interpretation. The refinement of physical prototypes from α (low fidelity) to Ω (high fidelity) is a known iterative sequence for designers. Analogously, model-based design follows a sequential paradigm.

2.1 Design Set Sizes
Set-Based Design, as described by Singer et al. [5], is an approach to the engineering design process that is specifically described in contrast to more traditional point-based design. The optimization community has used set-based methods as a means of expanding the trade space while leaving design solutions of varying kinds (e.g., rotary vane pump versus a peristaltic pump) in competition [10], introducing more designs and attributes into the trade space to compare/explore the space, and with non-homogeneous designs [11]. The premise is to maintain design freedom during preliminary stages and then intelligently reduce this freedom to permit fine-tuning later with minimal damaging effects from oversight of the restrictiveness in the design space [12].

A controlled convergence strategy’s efficiency, however, relies on a decision maker’s (DM’s) ability to eliminate inferior solutions rapidly and with conviction. Without this ability, a DM will continue to invest in empty ventures. Since this culling must be made with incomplete knowledge, traditional comparison methods are inappropriate [13]. The sequential process of design must then present a path to navigate multi-level modeling to reduce uncertainty and to achieve mandated model fidelity effectively.

2.2 Model Detail
According to systematic design by Pahl [14], conceptual design modeling takes requirements and objectives as inputs, and produces pertinent solution variables. Distillation of a problem into its primal form requires mapping of function spaces and attribute spaces and how these functions interact [15] with modeling fidelity bridging the space in between. Multi-fidelity modeling incorporates information progressively through stages that traverse a space of increasing analytic complexity and detail of design, and decreasing design set size.

An engineering configuration model takes as input overall attributes of the system (e.g., speed and range for an aircraft) along with technology options of interest (e.g., diesels and gas turbines for land vehicle propulsion) and produces output metrics. The configuration model can be expanded in breadth by including new technology options or by extending the range of validity of the overall attributes. It can be expanded in depth by increasing the modeling detail internal to the model and evolving the inputs and outputs of the model.

In terms of cost and time, the effort to extend the validity of the model in breadth is much less for lower fidelity concept models than it is for high fidelity models, and so the lower fidelity models help serve to narrow the region of interest prior to investing time and cost to develop the higher fidelity models [16]. Conceptual models undergo continual extensions in breadth as information is requested about new design options. This requires many small decision epochs since modeling a new design option takes time and money. The process of updating the models and running simulations is iterative until some agreeable level of fidelity and cost has been reached [7].

This desire to modify the model to support the search directly conflicts with the desire to have the highest fidelity and detail as possible, as a more detailed model is far harder to modify and validate for new trade space regions. This necessitates budgeting policies that often conflict with the goal of achieving satisfactory model fidelity in a finite and non-extendible schedule. The time allowed for model enhancement, iterative design space population, design space exploration, and culling, is always limited by resources and program schedules; so, the DMs must rapidly navigate the edge of analysis fidelity, design detail, and a variety of choices.

3. Choice as a Sequential Process
One approach to modeling decision making that is of particular utility to this effort has come from the domain of marketing, which is fundamentally concerned with how consumers make choices. While there are many different approaches to modeling the consumer’s choice process, a common thread is that the consumer goes through a process
of sequentially reducing the space of considered choices through a number of discrete sets. Shocker et al. [4] define a model that has been widely adopted in the field, which directly informs our efforts (see Figure 1). The model looks at the contraction of options from a universal set (set containing all alternatives) to the awareness (set of choices that have been enumerated) to the consideration and finally to the choice set (the final set that a DM would consider prior to selection). Although only one consideration set is shown, there can be many intermediate sets, each smaller than the predecessor while subjected to more scrutiny.

**Figure 1:** A model of individual choice showing convergence of problem space [4]

### 4. Choice Model Extended to Design

This model has now been extended by the authors to explicitly include the effort to use computational modeling to guide decision makers in their search through the trade space [7]. The basic premise is that decision makers and their team start with low fidelity (i.e., conceptual) designs subjected to low fidelity analyses to identify regions of interest and cull out regions that can be removed from further consideration, and then repeat while subjecting the reduced size set to further scrutiny. The final choice set is analyzed at maximum fidelity, and can in fact be considered to be the physical artifact itself exercised in field tests. Figure 2 shows an intermediate step in the larger convergence process.

In executing a trade space exploration and selection against a particular consideration set, at some point the DM makes the down-select to a smaller consideration set thus forming a decision epoch — their final decision is the single choice. The entire process can be viewed as a series of decision epochs, with each epoch incurring an allocation of time and resources resulting in a smaller consideration set for the next stage.

Figure 3 shows an example of the problem confronting a DM. It shows a sequence of modeling efforts and fidelities along with the sets carried forward. The DM must navigate this space, carrying a large enough set forward at each stage to ensure at minimum that an ultimately feasible solution is contained within the set, and ideally has the optimum from the entire space.

This description complements the structure resulting from literature in marketing [4] and adaptive decision making [17] research. Questions are then related to the rate at which the space of detail, accuracy, and set size of each model and how those models relate to one another leading to a final technical data package. Succinctly, a framework that purports to navigate the space of model detail, analysis complexity, and set reduction, must answer the following:
Figure 2: Trade space exploration and consideration sets

- At what rate must the level of detail in a design differ from previous epochs?
- What new analysis or accuracy/precision of analysis is required at the next epoch?
- To what degree should a designer be able to analyze, interrogate, and discern differences among solutions to reduce the available attribute space?

5. Preliminary Formal Model

This section presents a preliminary formal of the decision process, so that it may be analyzed and cast as an optimization problem in its own right. Start with a random list that represents the finite parameter trade space, $Z$, and where each element of $Z$ corresponds to a parameter of a design. An example would be spherical tank design where the three parameters are radius, wall thickness, and material choice and so $Z = Z(\text{radius, wall thickness, material})$ and each of the parameters can take values from some set of possibilities, continuous or discrete. If the three parameters could take 100 discrete values each, then $Z$ would have $100^3 = 1,000,000$ elements with an equal number of potential function evaluations.

The space of $Z$ has a probability measure over it, which reflects the probability that a particular point is the ideal point $z^\ast$. The ideal point is the design point that would be chosen if there were time and ability to analyze every point in $Z$ with the highest fidelity model. Initially, the probability is uniform across $Z$, reflecting a least informative prior as to the location of the ideal point.

We apply a “modeling effort” $M$ to $Z$ and this induces a new probability measure on $Z$ that refines and localizes the position of Image. The modeling effort is the realization of a decision epoch as in Figure 2. The modeling effort can be more or less comprehensive, with differing results for each case. So, if we have modeling efforts $M_1$ and $M_2$ where $M_2$ is more comprehensive, then using $M_2$ results in a more focused distribution as compared to using $M_1$.

Modeling efforts also have a cost associated. A simple assumption is that the cost of a modeling effort can be decomposed into a fixed cost for initially establishing an effort and a variable cost proportional to the size of the set $Z$ that the modeling effort is applied against. For example, if we use a linear approximation to cost, so cost $c_i$ of model $M_i$ is $\alpha_i + \beta_i |Z_i|$, where $|Z_i|$ is the size of the set $Z_i$. For purposes of modeling the process, the actual contents of $Z$ are not important, only the size of the set $Z$. Label the size of the set $|Z_i|$ as $q_i$, then:

$$c_i = \alpha_i + \beta_i q_i$$  \hfill (1)
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Figure 3: Fidelity progression from an initial low fidelity model, $M_1$, with input space $x_1$. A subset of the space is carried forward into the next fidelity model $M_2$, and the union of the spaces $x_1 \cup x_2$ and so forth until the highest fidelity model is developed, $M_n$. The sequence leads to a $\mathcal{Z}$ space that may or may not be feasible. This region may be suboptimal when compared to other regions to the refinements assumed during previous decision epochs.

The simple linear cost model neglects factors such as learning curves, where the cost per analysis should decrease with each analysis conducted. It also neglects sustainment costs merely for having a model in hand, such as annual licensing costs for software and hardware. These can be added to increase accuracy.

5.1 The Modeling Effort $M$

The modeling effort $M$ is all of the cost and time to define designs, to identify and acquire computational models, to bring these models up to the point of being able to use them, the time and effort to train operators of the tools, along with the cost and time to exercise the models, analyze the results, and down-select the trade space. The discriminatory power of the modeling effort is a function of both the tools used to execute the model and the people executing the effort. For example, the same toolset used by senior users empowered by the final decision maker would likely have more discriminatory power than a group of less trusted users. In summary, the modeling effort includes a technology element for populating the trade space and visualizing it and a human element for analyzing the results and making a decision, thus the model has a fixed and variable cost. Furthermore, the model has a discriminatory power which is reflected in its ability to localize the ideal point $z^\star$.

An important restrictive modeling assumption adopted for this preliminary work is that applying a model to a space reduces the space by a percentage of the original $q$, not by a percentage of the remaining $q_i$. Therefore, there are no gains to be realized by repeatedly exercising the same model. Similarly, the amount of reduction realized by running a model is independent of the set it is run over, unless the set’s size is smaller than $q_i$. This is a restrictive assumption that can be relaxed and modified in later models.

5.2 Single-Stage Modeling Versus Two-Stage Modeling

Look at the case where there are two potential efforts, $M_1$ and $M_2$, and assuming that $M_2$ is of the quality for which applying $M_2$ to all of $\mathcal{Z}$ will result in a new probability distribution that is singular, i.e., it will identify exactly $z^\star$. The effort $M_1$ will not identify the exact location of $z^\star$, but it will sharpen the distribution of $\mathcal{Z}$ and will further reduce the support of the distribution to some smaller region $\mathcal{Z}_i$ where $|\mathcal{Z}_i| < |\mathcal{Z}|$, or $q_i < q$.

If $M_1$ is run against $\mathcal{Z}$ first, then when $M_2$ is run, it need only be run over $\mathcal{Z}_1$ since we know that $z^\star$ does not lie in the set $\mathcal{Z} - \mathcal{Z}_1$. This results in a cost savings in running $M_2$ as the run cost depends on the size of the space to run it on. This is the fundamental key to executing the multi-stage modeling: the reduction in size of the set considered by the next stage of analysis is reduced from the efforts of the previous stage.

So, what must hold with regards to the costs of $M_1$ and $M_2$ and the reduction in support of the distribution by $M_1$ in
order to justify using a two-stage process? The cost of a single-stage process is \( c = \alpha_2 + \beta_2 q \). The cost for a two-stage process is then:

\[
c = \alpha_1 + \beta_1 q + \alpha_2 + \beta_2 q_1
\]

Solving for \( q_1 \) results in the following relationship

\[
q_1 \leq \frac{(\beta_2 - \beta_1)q - \alpha_1}{\beta_2}
\]  

So \( M_1 \) needs to restrict the space as Equation (3) in order to justify using it in a two-stage process.

5.3 Multi-State Process

The sequential process can have arbitrarily many modeling stages. Consider the case of having three potential models to employ. In this case, there are four potential configurations to consider (note that all four sequences end with \( M_3 \)):

\[
M_3
M_1M_3
M_2M_3
M_1M_2M_3
\]  

In general, if there are \( n \) potential models, then there is an upper bound of \( 2^{n-1} \) possible configurations to consider. This upper bound is loose, however, and can be tightened considerably.

Considering the alternatives, the cost to achieve full fidelity is taken from one of the following modeling paths:

\[
c_3 = \alpha_3 + \beta_3 q
\]

\[
c_{13} = \alpha_1 + \alpha_3 + \beta_1 q + \beta_3 q_1
\]

\[
c_{23} = \alpha_2 + \alpha_3 + \beta_2 q + \beta_3 q_2
\]

\[
c_{123} = \alpha_1 + \alpha_2 + \alpha_3 + \beta_1 q + \beta_2 q_1 + \beta_3 q_2
\]

The alternative paths can be laid out as a graph through a state space, with \( q_i \)’s as states and the models used as arcs in the graph.

![Figure 4: Sequencing of models](image-url)

Adopting the nomenclature \( M_iM_j \rightarrow M_{ij} \), the best modeling approach to get to state \( q_3 \) can be calculated as:

\[
M_{123} = \min [M_1M_3, M_{12}M_3, M_3]
\]  

while the best modeling approach to get to state \( q_2 \) is:

\[
M_{12} = \min [M_1M_2, M_2]
\]

and the only approach to get to \( q_1 \) is to use \( M_1 \). Of the possible \( 2^{n-1} \) configurations to choose from, one only needs to actually consider \( n + \ldots + 3 + 2 + 1 = (n^2 + n)/2 \) possible values, which can be written as:

\[
\text{number of combinations} = \binom{n+1}{2}
\]

The model can finally be formally described as a finite horizon sequential decision process:

- **States**: \( s \) from \( q_i \in \{0, n\} \)
- **Actions**: \( a \) from apply modeling \( M_i \), where \( i \in \{1, n\} \)
- **Costs**: \( c = c(q_i, M_j) \)
- **State transition**: for a state \( s_k = q_i \), and a modeling effort \( M_j \), the next state \( s_{i+1} \) is
To solve the problem one then needs to find a decision rule that chooses the least costly modeling effort to reach the final state. As an example, in Figure 5, we note that of a design set, the discriminatory power of a model of zeroth order has no discriminatory power and so the entire set of designs is still available. A step from analysis $i \rightarrow j$ incurs a cost but yields analysis information that helps reduce the size of the design space.

![Figure 5: Discriminatory power of a model of a specified analytic complexity. In this example, the highest fidelity analysis is $A_{10}$ and yields a single solution.](image)

### 6. Design Detail and Algorithmic Complexity in Modeling

The computation portion of the modeling process can be split into two complementary aspects that drive modeling cost and relate to key decisions; design detail and algorithmic complexity. Both can be controlled by the DM separately from each other, introducing a multiplicative increase in the number of potential modeling efforts. Figure 6 shows an example of distinct set of algorithmic efforts (horizontal) and design details (vertical) that are available to a decision maker who is tasked with conducting a hydrostatic analysis.

From a standpoint of time and cost, increasing the detail in a model takes the time for the designers to specify parameters form lower levels of detail, and geometry through CAD (for this example) as the detail increases. The jump to CAD can result in orders of magnitude increases in effort.

Similarly, the lower level algorithms may be fast and based on heuristics, while the highest fidelity ones may be very slow running, taking hours or days to compute results for a single design.

While it is in theory possible to run a high fidelity (and therefore costly) simulation using a low level of detail design as input, and similarly possible to create a very detailed design and then use low fidelity modeling assess it, the expectation, is that the path taken through the space will start in the upper left corner and in general proceed in increased analysis, increased design detail, and reduction in set size, ending up in the bottom-right. For the example in Figure 6, there are therefore $3 \times 3 = 9$ potential modeling efforts, 10 states $q$, $2^9 = 512$ potential choices of trajectory, and $\binom{10}{2} = 45$ ones to explore.

The addition of design detail into the space changes the problem into finding the most efficient trajectory through the space. There are many potential paths that can be utilized to reach the final fidelity with maximum model detail, maximum analytic complexity, and highest discriminatory power as shown nominally in Figure 7.

### 7. Examples of a Sequential Modeling Problem

Under DARPA’s Adaptive Vehicle Make portfolio of projects, Penn State is currently developing design tools to support manufacturability assessment for both conceptual and detailed designs with a test application to amphibious vehicles. We have also addressed hydrostatic and hydrodynamic-related performance measures such as stability and speed calculations. An example here is drawn from the hydrostatic and hydrodynamic calculations. The potential paths through the modeling/design/decision process are captured in Figure 6.
The two axes of the model are: (1) analysis tool complexity/fidelity and (2) level of design detail. Considering level of detail first, the design can be expected to start at a conceptual level, such as a simple list of key parameters, e.g., displacement, beam, draft, length at waterline and length overall, and progress to the complete detail embodied in a CAD file. Within CAD, the geometry can evolve in complexity, as shown in Figure 8.

The analysis tools similarly start with lower fidelity analyses, potentially using heuristics or simplified equations, and extend in complexity to higher fidelity models such as finite element analyses. The complexity of the analysis depends not only on the computational tool used, but also on the modeling within the tool. For example, the STAR-CCM+ is a hydrodynamic analysis which may incorporate different grid strategies with accompanying differences in time and cost to set up and run. Applying this work’s effort, each of the nine boxes defines a unique modeling effort and consisting of the tuple of design detail and analytic complexity.

A related example can be found in the subsequent use of the finite element method of a structure subject to probabilistic loading conditions. The design detail can be changed by adjusting the number of elements used to generate the discretized mesh. As the loading is probabilistic, Monte Carlo simulations are typically used and so the number of simulations performed prescribes the analytic complexity. The output metrics define the set size where terse analysis helps to remove poorer designs while retaining potential solutions.

8. Future Work

The work on developing a framework to navigate the triplet of design detail, analysis complexity, and set size under model-based design is preliminary and ignores dimensions of cost and uncertainty by necessity to facilitate understanding. The framework presents a novel way of investigating this space, and allows designers to move in the space discovering and learning along the way. Topics for further study include:

- Understanding rework, i.e., the implications of backtracking through previously utilized tuples
- Establishing metrics for identifying or suggesting trajectory policies
- Collecting empirical data on design processes using multi-fidelity models
Figure 7: Notional contours demonstrating that there are many paths through the space with differing number of epochs to reach the required state

Figure 8: Progression of CAD design detail

9. Conclusions

Complex engineered systems rely heavily on simulation models for design. This paper has provided background and information detailing how design is a sequential process that results in navigating a space triplet of model detail, analysis complexity, and set reduction. A simple framework for viewing the refinement in a sequential manner has also been presented. It is proposed that there are optimal paths from the lowest to the highest state and this can be viewed as an optimization process or optimal policy problem. The ability to traverse fidelities is of value since requirements and perceptions change and designs are viewed with varying degrees of preference. The framework is simple in structure but captures the key aspects of the sequential nature of design.

In practice, decision makers must decide when to stop expending resources and to select a solution. Arbitrary selection with poor or invalid information is dangerous and economics can become a weighting factor. The imprecision of design requires solution strategies that minimize risk. The proposed framework encourages designers to think about the process of refining their search for a satisficing design by investigating the sequential refinement of a simulatable space.

Acknowledgements

The authors wish to acknowledge the support provided by the Defense Advanced Research Projects Agency (DARPA/TTO) under contract HR0011-12-C-0075 iFAB Foundry for making this work feasible. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of DARPA or the U.S. Government.
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