An optimization model for the design of rack storage systems

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Abstract

The paper deals with the design of rack storage systems in multiple product situations. Given the production/delivery patterns of the different products and shared storage policies, the aim is to identify the amount of storage area that should be devoted to single deep selective racks and the amount for non-selective racks. The non-selective rack storage systems under analysis are accessed in a LIFO (last-in first-out) manner (e.g., “drive-in” racks). In particular, the racks under analysis consist of lane levels of different heights. This makes the problem challenging when also the unit loads may have different heights (e.g., because of product load restrictions). In such a situation, small unit loads can be put in high lanes, not vice versa. Moreover, the volumetric utilization and the storage efficiency of the warehouse become key performance indicators, so that the optimal mix of racks of different heights should be investigated. Thus, the paper presents a mathematical programming model able to address the considerations outlined above, along with floor space constraints. The objective is identify the number of single deep selective racks, the mix and number of non-selective racks and the lane depths so that the volumetric storage efficiency is maximized.

Keywords
Optimization model, rack storage systems, optimal lane depth, volumetric storage efficiency, multiple product.

1. Introduction

Warehouse design is one of the most important and challenging problems in the manufacturing sector. In [1, 2] the authors identify five major decisions concerning warehouse design: (1) determining the overall warehouse structure; (2) sizing and dimensioning the warehouse and its departments; (3) determining the layout within each department; (4) selecting warehouse equipment; and (5) selecting operational strategies. This work is about the assignment of unit loads of different heights and types of products to different types of rack storage systems. So, the focus is on the third of the categories introduced above.

The optimal storage location assignment is generally addressed by considering material handling costs and space utilization [see, e.g., layout studies such as 3-6]. In particular, in [4] a dynamic programming algorithm has been developed to identify the optimal lane depths in block stacking storage systems in order to maximize space utilization. Nevertheless, when the unit loads have different heights, the volumetric utilization becomes more significant than space utilization. Moreover, in [7] the authors propose a new performance measure, called storage efficiency, which considers the impact of the last-in first-out (LIFO) retrieval policy on the performance of the rack storage system. Then, a heuristic algorithm is proposed to allocate a set of product types to “drive-in” racks which may have different lane depths, by considering the resulting storage efficiency. However, the following limitations can be noted: (1) no exact solution is available, and (2) the volumetric utilization is not taken into consideration because the unit loads in the warehouse have the same height and the drive-in racks have the same number of levels and the same usable height per level.

This paper proposes a mathematical programming model for addressing a multiple product situation where the unit loads may have different heights. In particular, the objective of this study is to identify the number of selective racks, the mix and number of non-selective LIFO racks and the lane depths so that both the volumetric utilization and the storage efficiency are taken into account.
1.1 Problem statement

In this paper a multiple product situation is considered. In particular, we assume that different products may have different attributes such as size, shape, and weight. These products are palletized onto standard pallets (e.g., EUR-pallets) so as to obtain the unit loads that the warehousing system has to handle. Each unit load involves a single type of product and unit loads of the same product have the same size. On the other hand, unit loads of different products may have different heights. This can be due to a single or combination of reasons such as:

- pallet loading constraints (items must be palletized without exceeding either the pallet maximum height or the maximum weight, so that the most restrictive constraint determines the actual height of the unit load);
- transportation issues (payload constraints are to be strictly observed);
- customers’ requests (some customers specify the size of the unit loads they are willing to receive).

Hence, the warehousing system under analysis has to handle unit loads of different products with different heights, and this directly affects the design of the rack storage system. In particular, the rack storage system addressed in this paper can include:

- non-selective LIFO racks such as “drive-in”: this type of racks generally requires less floor space (a single access-point for both put-away and retrieval operations), but the unit loads are not independently accessible;
- selective racks: this type of racks ensures that products are rotated properly, but they are generally more costly and space-consuming than LIFO racks.

For further discussion about advantages and disadvantages of rack storage systems, the reader may refer to [8].

For the sake of clarity, Figure 1 shows a drive-in rack with lanes that are 5-pallet deep and 3-level high so that each of them has a total capacity of 15 unit loads.

The height of each lane level depends on the position of the guide rails. For a given lane, we assume that all the levels have the same height and that the position of the guide rails cannot be easily changed once installed. Thus, different types of racks can be considered, according to the height and, consequently, the number of the lane levels.

As an example, in the case study discussed in this paper two types of drive-in racks have been considered: (1) racks whose levels have a usable height of 1.7 m (designed for unit loads that are 1.6 m tall); and (2) racks whose levels have a usable height of 1.3 m (designed for unit load that are 1.2 m tall). The internal usable height of the warehouse is such that: (1) racks of the first type have four levels; (2) racks of the second type have five levels. Thus, lanes of the first type are more flexible (i.e., suit unit loads of any type of product). On the other hand, if the depth is the same, lanes of the first type have a lower capacity than lanes of the second type with a greater number of vertical levels.

Selective racks have a depth of a single pallet location resulting in a greater versatility and selectivity. In this paper we assume that the levels of selective racks have the same usable height, the maximum height of the levels of LIFO racks. Thus, selective racks are suitable for all types of products.

The aim of the present study is to design the rack storage system discussed above by determining:

- for non-selective LIFO racks, the number of lanes that are to be installed in the warehouse, by specifying the height of the levels (i.e., the type of racks) and the depth (in number of pallet footprints).
- for selective racks, the number of pallet locations that are to be arranged in the warehouse.

The objective is to maximize the volumetric utilization of the storage system by ensuring that all the unit loads can be properly stored in the warehouse.

The different products in the system under analysis are assumed to have similar production/delivery patterns. In particular, their behavior is characterized by:

- cyclic production: when a batch of a certain product starts, the production lasts for a given production time at a constant production rate (depending on the type of product). Then, the production is interrupted for a period, called non-production time, until a new batch of that product starts again.
- continuous delivery: the unit loads leave the storage area at a continuous and constant rate.

The sum of the production and non-production times is called cycle time. Different products may have different cycle times with different production and demand rates. The inventory curve over time presents a linear increasing-decreasing trend as shown in Figure 2.

Finally, shared storage policies are adopted for the products under analysis.
The remainder of the paper is organized as follows. Section 2 provides the notation used throughout this paper. Section 3 discusses the definition and computation of a performance measure introduced by \[7\] and called storage efficiency. This performance measure, along with other issues introduced in this section, enters in the formulation of a 0−1 programming model. This model is described in Section 4. Section 5 illustrates a case study where the proposed original model has been applied. Finally, conclusions and suggestions for further research are presented.

2. Notation

- \(j = 1, \ldots, J\): different products whose unit loads are to be stored in the warehouse;
- \(t = 1, \ldots, T\): types of non-selective LIFO racks. Lanes of different rack types have levels of different heights;
- \(r = 2, \ldots, R\): depths (in number of pallet locations) of non-selective LIFO racks. Note that \(r > 1\) (\(r\) is equal to one for selective racks only);
- \(w, d\): width and depth (in meters) of the unit loads (unit loads may only differ in height);
- \(W^d, D^d\): width and depth (in meters) of a single pallet location of non-selective LIFO racks (including the necessary gaps between unit loads and rack frame);
- \(W^s, D^s\): width and depth (in meters) of a single pallet location of selective racks (including the necessary gaps between unit loads and rack frame);
- \(h_j\): height (in meters) of a unit load of product \(j\);
- \(H^d_t\): height (in meters) of the levels of lanes of rack type \(t\);
- \(H^s\): height (in meters) of the levels of selective racks. We assume that \(H^s = \max \{H^d_t\}\);
- \(z^d_t\): number of levels of lanes of rack type \(t\);
- \(z^s\): number of levels of selective racks;
- \(A^d\): width (in meters) of any aisle serving non-selective LIFO racks;
- \(A^s\): width (in meters) of any aisle serving selective racks;
- \(S_{\text{tot}}\): storage area (in square meters) available in the warehouse;
- \(p_j, r_j\): production and demand rate, respectively, of product \(j\) (in unit loads per day);
- \(T_{C_j}, T_{P_j}\): cycle time and production time (in days) of product \(j\) (where the production time is included in the cycle time);
- \(s_j\): safety stock of product \(j\) (in unit loads);
- \(I^M_j, I_j\): maximum and average inventory level, respectively, of product \(j\) (in unit loads). Thus, \(I^M_j = s_j + (p_j - r_j)T_{P_j}\) and \(I_j = s_j + I^M_j/2\);
- \(I^\text{max}_j\): maximum storage period of product \(j\) (in days);
- \(\alpha^*\): storage flexibility parameter;
- \(e_{jr}\): storage efficiency of product \(j\) when its unit loads are stored in lane of rack type \(t\) \(r\)-pallet deep (computed according to Section 3).
3. Storage efficiency and average number of racks

According to [7] we define storage efficiency a performance measure equals to 1 for selective racks and less than 1 for non-selective LIFO racks such as drive-in racks.

This is because when the first unit load of a certain product is placed in a lane of non-selective LIFO racks, the whole lane must be devoted to that product until it will be completely empty again. Thus, when the first pallet location is occupied the other locations result to be “constrained” for the time required to empty the lane. These replenishing/emptying cycles, typical of LIFO racks, depend on both the inventory profile of the stored product and its maximum storage period. The latter aspect is particularly relevant for perishable goods such as food, chemicals, and pharmaceutical products. If the time required to empty a lane (called emptying interval in the following) exceeds the maximum storage period, the product should not be stored in that lane.

In this section we deal with the storage efficiency and the average number of lanes required to store a set of different products whose behavior is characterized by cyclic production and continuous delivery (refer to Section 1.1 and Figure 2).

3.1 Drive-in Storage

In this section we briefly discuss the method proposed by [7] to compute the storage efficiency in case of non-selective LIFO racks.

The idea is to divide the cycle time of a certain type of product into emptying intervals of the same length, where an emptying interval is the time required for a lane of a certain type and depth to become empty, given that it is initially full. Then, for each emptying interval the number of required lanes of that type and depth is computed. Finally, the required lanes are averaged over the cycle time of the product under analysis.

Thus, for a product $j$ to be stored in lanes of rack type $t$, $r$-pallet deep (i.e., with a capacity $z_{tr}$) the length of the emptying intervals is:

$$l_{tr} = \frac{z_{tr}}{d_j}.$$
The number of emptying intervals in the cycle time can be expressed as:

\[ K_{jtr} = \left\lfloor \frac{T_{Cj}}{l_{jtr}} \right\rfloor. \quad (2) \]

As an example, in Figure 2 the cycle time is divided into five emptying intervals.

Then, for each emptying interval \( k \), with \( k = 1, \ldots, K_{jtr} \), the number of required lanes \( N_{jtr}(k) \) depends on the maximum inventory level during that emptying interval and denoted by \( \bar{I}_{jtr} \). In particular, using the notation defined in Section 2 we have:

\[
j_{jtr}^M(k) = \begin{cases} 
  s_j + (p_j - d_j)l_{jtr} & \text{if } k = 1, \\
  s_j + (p_j - d_j)l_{jtr} & \text{if } kl_{jtr} < T_{P_j}, \\
  s_j + (p_j - d_j)l_{jtr} & \text{if } (k-1)l_{jtr} < T_{P_j}, \\
  s_j + (p_j - d_j)l_{jtr} & \text{if } (k-1)l_{jtr} \geq T_{P_j}, \\
\end{cases} \quad (3)
\]

So, the number of lanes of rack type \( t \), \( r \)-pallet deep, necessary to store product \( j \) during the emptying interval \( k \) (with \( k = 1, \ldots, K_{jtr} \)) is determined as:

\[ N_{jtr}(k) = \left\lfloor \frac{j_{jtr}^M(k)}{r} \right\rfloor. \quad (4) \]

Then, averaging over the cycle time, we obtain the average number of lanes of rack type \( t \) \( r \)-pallet deep necessary to store product \( j \):

\[ \bar{N}_{jtr} = \frac{\sum_{k=1}^{K_{jtr}} N_{jtr}(k) + (T_{Cj} - (K_{jtr} - 1)l_{jtr})N_{jtr}(K_{jtr})}{T_{Cj}}. \quad (5) \]

Note that, if the cycle time is not an integer multiple of the length of the emptying intervals, the last emptying interval should be considered only for the portion \( T_{Cj} - (K_{jtr} - 1)l_{jtr} \).

Finally, for each product \( j \) stored in \( r \)-pallet deep lanes of rack type \( t \) we can compute the storage efficiency as the ratio between the number of pallet locations occupied to the number of pallet locations dedicated to that product during the cycle time. Thus, we have:

\[
\epsilon_{jtr} = \frac{2T_{Cj}s_j + T_{Cj}((P_t^M - s_j))}{2z_{jtr}r(\sum_{k=1}^{K_{jtr}} N_{jtr}(k) + (T_{Cj} - (K_{jtr} - 1)l_{jtr})N_{jtr}(K_{jtr}))}. \quad (6)
\]

### 3.2 Selective Storage

The storage efficiency in case of selective racks is equal to one by definition. This is because each unit load is independently accessible.

In [7] the authors compare selective racks and non-selective LIFO racks by identifying a threshold level for the storage efficiency. If the storage efficiency, obtained according to Equation 4, is higher than this threshold level the considered LIFO rack system performs better than the selective system. Otherwise, it would be more convenient to adopt the selective storage solution.

The threshold level can be computed by considering the space utilization for the two types of rack storage systems. Recalling the notation defined in Section 2 the space utilization for selective racks can be computed as

\[ u^s = \frac{wd}{(D^s + A^s/2)W^s}. \quad (7) \]

for non-selective LIFO racks type \( t \) \( r \)-pallet deep, it can be expressed as

\[ u^d = \frac{rd^2}{(D^d + A^d/2)W^d}. \quad (8) \]

Since the number of vertical levels for selective racks is \( z^s \) while for LIFO racks is \( z^d \) (depending on the rack type \( t \)), we can estimate the threshold levels for the storage efficiency of LIFO racks type \( t \) \( r \)-pallet deep as follows:

\[ e_{jtr}^L = \frac{u^s z^s}{u^d z^d}. \quad (9) \]

Thus, suppose that we are evaluating the allocation of product \( j \) to lanes of rack type \( t \) \( r \)-pallet deep. If we find that \( e_{jtr} < e_{jtr}^L \), it is more convenient to allocate that product to selective racks.
4. Optimization Model

The decision variables of the model are the set of binary variables \( x_{jtr} \), stating if product \( j \) is stored in drive-in lanes of rack type \( t \) with a depth of \( r \) pallets, and the set of binary variables \( y_j \), stating if item code \( j \) is stored in selective racks. Recall that selective racks have levels of the same usable height. In particular, selective racks are such that they are suitable to hold unit loads of all the types of products.

Thus, we have:

\[
x_{jtr} = \begin{cases} 
1 & \text{if item } j \text{ is stored in lanes of rack type } t, r\text{-pallets deep} \\
0 & \text{otherwise} 
\end{cases} \quad \forall j, t, r, \tag{10}
\]

\[
y_j = \begin{cases} 
1 & \text{if item } j \text{ is stored in selective racks} \\
0 & \text{otherwise} 
\end{cases} \quad \forall j, t. \tag{11}
\]

We assume that each product can be stored either in non-selective LIFO racks or in selective racks. If a product is stored in non-selective racks, it is assigned to lanes of the same type and depth. Thus, for a given product \( j \) one and only one decision variable is set to 1.

The objective function, that should be maximised, is the sum of the volumetric utilizations of each product in the warehouse. Thus, it can be expressed as:

\[
f(x_{jtr}, y_j) = \sum_j \sum_t \sum_r \left( \frac{r}{D^d + A^d/2} \right) W^d H^r_0 e_{jtr} x_{jtr} + \sum_j \left( \frac{d}{2} \right) \bar{w}_d \bar{h}_j y_j. \tag{12}
\]

Note that \( f(x_{jtr}, y_j) \) explicitly depends on the storage efficiency \( e_{jtr} \), computed according to Equation (6) for non-selective racks and equal to 1 for selective racks.

Using the above definitions, the 0 − 1 mathematical programming model is formulated as follows:

\[
\max f(x_{jtr}, y_j)
\]

s.t.

\[
\sum_j \sum_r x_{jtr} \leq 1, \quad \forall j \tag{13}
\]

\[
y_j = 1 - \sum_j \sum_r x_{jtr}, \quad \forall j \tag{14}
\]

\[
\sum_j \sum_r \left( r D^d + A^d/2 \right) W^d H^r_0 x_{jtr} + \sum_j \left( D^s + A^s/2 \right) W^s H^s y_j \leq S^{tot}, \tag{15}
\]

\[
x_{jtr} \leq 1, \quad \forall j, t, r \tag{16}
\]

\[
x_{jtr} \leq 1 + \frac{e_{jtr} - e_T}{e_T}, \quad \forall j, t, r \tag{17}
\]

\[
x_{jtr}, y_j \in \{0, 1\}, \quad \forall j, t, r \tag{18}
\]

Constraint set (13) guarantees that if a product \( j \) is assigned to non-selective LIFO racks, it is stored in lanes of the same type and depth. If product \( j \) is not assigned to non-selective LIFO racks, it must be assigned to selective racks (as stated by constraint set (14). Constraint (15) guarantees that the resulting area occupied by the rack storage system does not exceed the available storage area of the warehouse. Constraint set (16) prevents the assignment of any product to non-selective lanes if the emptying interval (defined according to Equation (11)) exceeds the maximum storage period of that product. In other words, the unit loads of a certain product must be retrieved before the maximum storage period expires. Constraint set (17) prevents the assignment of a product to non-selective LIFO racks when selective racks are more convenient (refer to Section 3 for details about the definition of the storage efficiency \( e_{jtr} \) and its threshold level). Constraint set (18) ensures that the decision variables are integer and zero-one.

4.1 Storage flexibility

As discussed in Section 1.1, both the unit loads and the lanes levels may have different heights. In the case study discussed in this paper we can distinguish between:
• unit loads that are 1.6 m tall;
• unit loads that are 1.2 m tall.

Similarly, as regards non-selective LIFO racks, we can distinguish between:
• racks whose lanes can hold both kinds of unit loads since the levels have a usable height of 1.7 m. Each lane have 4 levels. These racks are identified as racks type 1;
• racks whose lanes can hold only unit loads 1.2 m tall. Each lane have 5 levels. These racks are identified as racks type 2.

We assume that the selective racks are such that they can hold any type of product (i.e., the usable height is the same as racks type 1).

If the warehousing system were composed of racks type 1 only, all the unit loads would be stored in any lane. Thus, the storage flexibility would be maximized. On the other hand, the volumetric utilization would be negatively affected by the fact that all the racks have 4 levels only.

Thus, it is convenient to introduce a new constraint that can be formulated as follows:
\[
\frac{\sum_j \sum_r r z_{t=1} N_{j,t=1,r} x_{j,t=1,r} + \sum_j I_j y_j}{\sum_j \sum_r r z_{t=1} N_{j,r} x_{j,r} + \sum_j I_j y_j} \geq \alpha^*.
\] (19)

Constraint (19) ensures that the ratio of the number of pallet locations in racks type 1 (more flexible) to the total number of pallet locations must be higher than a certain parameter \(\alpha^*\). By varying \(\alpha^*\) we obtain solutions that differ from one another by the level of flexibility of the racks storage system and, consequently, by the volumetric utilization.

We expect that the two performance measures, i.e., volumetric utilization and flexibility, move in opposite directions so that as the number of lanes of rack type 1 increases with respect to the optimal solution obtained by relaxing constraint (19), the volumetric utilization of the warehousing system decreases. Thus, by varying the parameter \(\alpha^*\) different trade-offs between volumetric utilization and storage flexibility can be investigated.

4.1.1 Linearized model with storage flexibility

Constraint (19) is not linear. Hence the model with storage flexibility formulated as
\[
\max_f (x_{jtr}, y_j)
\]
s.t.
\[
\text{(13)} - \text{(18)}
\]

is a non-linear model.

Thus, it is convenient to modify constraint (19) as follows:
\[
\frac{\sum_j \sum_r r z_{t=1} N_{j,t=1,r} x_{j,t=1,r} + \sum_j I_j y_j}{\sum_j \sum_r r z_{t} N_{j,r} x_{j,r} + \sum_j I_j y_j} \geq \alpha^*,
\] (20)

where \(P\) is the total number of pallet locations dedicated for storing all the products in the optimal solution.

Since \(P\) is unknown, its value in the linear constraint (20) can be initially set to a lower bound. A possible lower bound on the number of pallet locations \(P\) can be defined as follows:
\[
\text{LB}^P = \sum_j I_j.
\] (21)

This is because the minimum number of pallet locations dedicated to the products in the warehousing system is the sum of the average inventory levels. This represents the situation where selective racks only (with a storage efficiency equal to 1) are available in the warehouse.

Then, a recursive procedure based on the modified linear model is developed in order to find an admissible solution to the non-linear model with storage flexibility. The steps of the proposed recursive procedure are as follows:

**Step 0** Set \(i = 1\). Calculate a lower bound \(\text{LB}^P\) on the value of \(P\) (e.g., according to Equation (21)) and set \(P^{(0)} = \text{LB}^P\).

**Step 1** Apply the following \(0 - 1\) linear model:
\[
\max_f (x_{jtr}, y_j)
\]
s.t.
\[
\text{(13)} - \text{(18)} \text{ and (20)}
\]

and find the actual solution \(x^{(i)}_{jtr}\) and \(y^{(i)}_j\).
Step 2 Calculate the actual number of pallet locations as

\[ P^{(i)} = \sum_j \sum_r r z_j N_j r^{(i)} + \sum_j I_j N_j r^{(i)} \]

If \( P^{(i)} = P^{(i-1)} \) then go to Step 4; else go to Step 3.

Step 3 Let \( i = i + 1 \). Go to Step 1.

Step 4 STOP.

5. Case Study

The optimization method proposed in the present paper has been applied to a real instance taken from a warehousing system located in Emilia Romagna Region (Italy) and operating in the food industry.

A set of one hundred different product codes have been analysed (i.e., \( j = 1, 2, \ldots, 100 \)). As introduced in previous sections, items are palletized so that we can distinguish between: (1) unit loads that are 1.6 m tall and (2) unit loads that are 1.2 m tall. In particular, 20 product codes are palletized according to the first standard, and the remaining 80 product codes according to the second standard. Historical data for the production and delivery flows of the different products have been analysed in order to compute the required input parameters (such as production/demand rates, cycle time, production time, safety stock, maximum and average inventory levels). In particular, the total average inventory level is 2340 [units/day] for unit loads that are 1.6 m tall (i.e., 17% of the total average inventory) and 11270 [units/day] for unit loads that are 1.2 m tall (i.e., 83% of the total average inventory).

The rack storage system can include: (a) selective racks; (b) drive-in racks of type 1 (\( t = 1 \)) suitable for unit loads of any product code; (c) drive-in racks of type 2 (\( t = 2 \)) suitable only for unit loads 1.2 m tall. We assume that selective racks are served by turret trucks (allowing short side pallet handling) so that the aisle width \( A_s \) is 1.8 m. Non-selective racks are served by reach trucks which require a larger aisle width \( A_d \) of about 3.3 m. The remaining dimensional parameters for selective racks are reported in Table 1, for drive-in racks in Table 2. The lanes of the drive-in racks have a depth that can vary from 3 pallet locations to 15 pallet locations (i.e., \( r = 3, 4, \ldots, 15 \)).

For each product \( j \), rack type \( t \) and lane depth \( r \) the storage efficiency \( e_{j tr} \), the threshold level \( e_{tr}^T \), and the average number of lanes \( \bar{N}_{j tr} \) have been computed according to Section 3.

Table 1: Selective racks: dimensional parameters

<table>
<thead>
<tr>
<th></th>
<th>( D_s ) [m]</th>
<th>( W_s ) [m]</th>
<th>( H_s ) [m]</th>
<th>( z_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SELECTIVE racks</td>
<td>1.3</td>
<td>0.9</td>
<td>1.7</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2: Drive-in racks: dimensional parameters

<table>
<thead>
<tr>
<th></th>
<th>( D_d ) [m]</th>
<th>( W_d ) [m]</th>
<th>( H_d ) [m]</th>
<th>( z_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRIVE-IN racks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>type 1</td>
<td>0.9</td>
<td>1.3</td>
<td>1.7</td>
<td>4</td>
</tr>
<tr>
<td>type 2</td>
<td>0.9</td>
<td>1.3</td>
<td>1.3</td>
<td>5</td>
</tr>
</tbody>
</table>

Then, the optimization model described in Section 4 has been applied using a standard programming solver.

First, the linear programming model without the storage flexibility constraint has been solved. Table 3 shows the mix of racks and the number of lanes that should be installed in the storage area. This solution implies a volumetric utilization of 47.1% for drive-in racks and 44.0% for selective racks.

Then, the storage flexibility has been taken into account by using the recursive procedure described in Section 4. The solutions for two possible values of the parameter \( \alpha^* \) are reported in Table 4. As expected, as the value of \( \alpha^* \) increases, the number of lanes of rack type 1 increases as well, allowing the warehousing system to have an higher flexibility in terms of capability of accepting different types of unit loads. It can be noted that by passing from \( \alpha^* = 0 \) (i.e., case without the flexibility constraint) to \( \alpha^* = 0.2 \) the number of selective racks does not change. An increase in the number of selective racks occurs only when \( \alpha^* = 0.3 \). The reason why this occurs is that it is convenient to first invest on non-selective racks in order to avoid to negatively affect the volumetric utilization. In particular, if \( \alpha^* = 0.2 \) the volumetric utilization is 46.9% for drive-in racks and 44.0% for selective racks; if \( \alpha^* = 0.3 \) the volumetric utilization is 46.6% for drive-in racks and 42.8% for selective racks.
Table 3: Optimal solution (without storage flexibility constraint)

<table>
<thead>
<tr>
<th>Rack Storage System</th>
<th>Depth $r$</th>
<th>Num. of Lanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRIVE-IN type 1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
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| SELECTIVE           |           | **255**       |

6. Conclusions
This study has provided a mathematical programming model for determining (1) the number of single deep selective racks; (2) the mix and number of non-selective LIFO racks (of different lane depths), to install into a warehousing system with different types of unit loads (in terms of product and size). Since both the lane levels and the unit loads may have different heights two performance measure, i.e., volumetric utilization and storage efficiency, should be jointly taken into account. Moreover, the storage flexibility, expressed in in terms of capability of accepting different types of unit loads, can be included as an additional constraint.

The analytical approach described in this paper can be extended in order to consider other types of storage systems (e.g., gravity rack system, pallet shuttles, etc.). Then, further research should be conducted into methods of incorporating the proposed approach into automatic layout generators.

References
Table 4: Optimal solution with storage flexibility

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