Positioning Automated Guided Vehicles in a General Guide-Path Layout

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Abstract

The locations of dwell points for idle vehicles in an automated guided vehicle (AGV) system determine the response time for pick-up requests and thus affect the performance of automated manufacturing systems. In this research, we address the problem of optimally locating dwell points for multiple AGVs in general guide-path layouts with the objective of minimizing the mean response time in the system. We propose a mixed integer linear programming (MILP) formulation for the problem. We also develop a generic genetic algorithm (GA) to solve the problem. The MILP models and the GA procedure are illustrated using a two-dimensional grid layout problem. Our computation study results show that the proposed GA procedure can yield near optimal solutions for these test problems in reasonable time.

Keywords: Automated Guided Vehicle System; Dwell Point Location; Mixed Integer Linear Programming Model; Genetic Algorithm;

1. Introduction

An AGV is a driverless vehicle that follows wires in the floor, or uses vision or lasers. AGV systems are most often used in industrial applications to move materials around a manufacturing facility or a warehouse. They have been implemented in a large variety of industries such as aerospace, automotive, chemical, electronics, plastic, food and beverage, and textiles as well as in inter-modal container ports. AGV system implementations could potentially lead to better production planning and control, safety, cost reduction, and flexibility. AGVs also offer a seamless interface with increasingly used automated warehousing systems, which can be controlled by integrated warehousing management techniques. However, these potential benefits can only be obtained when adequate guide-path layouts and control systems for vehicle dispatching, routing, and traffic management are available.

Logistics problems in the implementation of an AGV system comprise location of pick-up and delivery (P/D) points, optimal guide-path design, determining the optimal number and type of AGVs, positioning of AGVs, assignment of AGVs to pick-up requests, routing and dispatching of AGVs, and resolution of deadlocks and routing conflicts. Among these problems, positioning of AGVs and assignment to pick-up requests is an important control issue in AGV system implementation ([1]). Since the workload of a manufacturing system changes over time, the idleness of the material handling equipment used in the facility is expected in order to avoid system overload. Thus, in an AGV system, a relevant problem is to decide the location of an AGV when it finishes a delivery job and has no immediate assignments, because the position of the idle vehicle determines the empty travel time to the next pick-up request, which is called the response time. The reduction of response times contributes to the reduction of the overall material handling time and this is an important objective in the material handling design process. In manufacturing

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environments such as job shops, minimizing the mean response time is the prevalent operational performance measure for AGV systems because it helps to reduce the overall material handling cost ([2], [3]).

Kim [2] proposed an algorithm to determine the optimal location of a new vehicle in a single loop guide-path layout with the objective of minimizing the mean response time when some idle vehicles are already parked at some locations in the layout. He considered both the uni-directional and bi-directional guide-path layouts. Chang and Egbelu [4] considered the dwell point location problem for a single AGV in the single loop guide-path layout, where the materials to be picked up in a station change dynamically over time with the objective of minimizing the expected response time of the vehicle. Lee and Ventura [3] considered the single loop guide-path layout and developed a polynomial time DP algorithm for optimally locating multiple dwell points for AGVs that minimize the mean response time. Both uni- and bi-directional guide paths were considered. Kim and Park [5] evaluated different idle vehicle circulation policies in a semi-conductor fabrication line with the objective of minimizing the average lead time. Based on a simulation study, they concluded that idle vehicle circulation policy significantly affects the efficiency of the system.

In recent years, general guide-path layouts with cross-aisle and angled-aisle structure, which are special cases of grid type layouts, are becoming more prevalent in manufacturing industries and warehouses in order to utilize space more efficiently and to reduce travel times between locations. Examples of guide-path layout topologies include the circle or loop layout, tandem loop layout, uni-dimensional grid layout, and two-dimensional grid layout (see Figure 1). To our knowledge, there is no publication that considers locating idle AGVs in general guide-path layouts. This study addresses the issue of optimally determining dwell points for multiple AGVs in general guide-path layouts, which can be modeled as directed networks. We consider the objective of minimizing the mean response time of the system.

![Examples of general guide-path layout topologies for AGV systems](image)

**Figure 1.** Examples of general guide-path layout topologies for AGV systems [8]

The remainder of this paper is organized as follows: In Section 2, we describe the dwell point location problem and develop a mixed integer linear programming (MILP) model for the problem. In Section 3, we propose a generic genetic algorithm (GA) to solve the problem for minimizing any linear/non-linear function of the response times. In Section 4, we present an illustrative example to show the application of the proposed mathematical model and GA procedure. Section 5 presents some conclusions on the work.

2. Proposed dwell point location model for general guide-path layouts

In this section, we present a MILP model to optimally locate dwell points for AGVs in general guide-path layouts, which can be characterized as directed networks. In a uni-dimensional grid layout, all AGVs move in the same
direction either clockwise or counter-clockwise, while in a two-dimensional grid layout, vehicles can change direction depending on the arc they are traversing. The underlying topology of an AGV guide-path can be modeled as a general directed network $G=(V,A)$, where $V$ is the set of nodes and $A$ comprises the arcs defined by pairs of nodes in $V$. An intersection point can be defined as a point where at least three guide-path segments meet, which may or may not coincide with a pick-up and drop-off (P/D) station. Note that, while circular or loop layout do not have intersection points, uni-dimensional and two-dimensional grid type layouts must have at least one. A node of $G$ may be a P/D station, an intersection point, or an intersection point with a P/D station. Let $I=\{v_1,v_2,\ldots,v_m\}$ be the set comprising the $m$ P/D stations. Let $Q=\{y_1,y_2,\ldots,y_q\}$ be the set containing the $q$ intersection points in the guide-path. Then, $V=I\cup Q$. Note that, in well-designed guide-path system, the out-degree of all nodes of $G$ must be at least 1. Otherwise, a node with an out-degree of 0 would create a deadlock state.

Let $J=\{x_1,x_2,\ldots,x_n\}$ be a set of dwell points in $G$ where the $n$ AGVs are positioned when they become idle. In addition, let $S=\{S_1,S_2,\ldots,S_n\}$ be a partition of set $I$, where $S$ includes the P/D stations served by the AGV assigned to dwell point $x_i$. Note that a pair $(J,S)$ provides a feasible solution to the dwell point location problem as long as the dwell points in $J$ belong to $G$, $S$ is a partition of $I$, and vehicle time restrictions are satisfied.

Let $d(x_j,v_i)$ be the shortest distance from dwell point $x_j$ to pick-up station $v_i$, for $x_j\in J$, $v_i\in I$. It is assumed that $d(x_j,v_i) = 0$ if and only if $v_i = x_j$; otherwise, $d(x_j,v_i) > 0$. We use the Floyd–Warshall algorithm ([6]) to determine the lengths of the shortest paths between dwell points and P/D stations. Let $s_g$ be the speed of a vehicle when it travels empty and $r_g = d(x_j,v_i)/s_g$ be the response time from dwell point $x_j$ to pick-up station $v_i$. The response time $r_g$ for a request from P/D station $v_i$ handled by an AGV located in dwell point $x_j$ is the empty travel time from $x_j$ to $v_i$. We partition the set of intersection points in $Q$ into $Q$ and $Q'$, where $Q$ is the set of intersection points in $G$ with out-degree equal to 1 and $Q'$ is the set of intersection nodes in $G$ with out-degree of at least 2. Thus, $Q = Q \cup Q'$ and $V = I \cup Q \cup Q'$. In addition, let $f_g(J, S)$ be the objective function for the dwell point location problem representing the regular performance measure of response times. Thus, $f_g(J, S)$ is a non-decreasing function of the response times $\{r_{ij} \mid x_j \in J, v_i \in S_j\}$.

Ventura and Rieksts [7] proved that there exists an optimal set of dwell points that minimizes the maximum response time in a loop guide-path layout, where all dwell points coincide with P/D station locations. [8] show that for any regular performance measure of response times $f_g(J, S)$ for an AGV system with $n$ vehicles and $m$ P/D stations in a general guide-path layout, there exists an optimal solution $(J^*, S^*)$, where dwell points are either P/D station locations or intersection points with out-degree of at least 2, i.e., $J^* \subseteq I \cup Q'$ (see Theorem 1 in [8]).

**Problem (P): Minimizing the mean response time ([3])**

The objective of this problem is to optimally determine the dwell points for idle AGVs that minimize the mean response time in a general guide-path layout.

Based on Theorem 1 in [8], $I \cup Q' = \{x_1, x_2, \ldots, x_p\}$ is the set of potential dwell points for positioning idle AGVs, where $p = m + q'$ is the cardinality of $I \cup Q'$ and $q'$ is the number of intersection points with out-degree of at least 2.

Let $f_{ik}$ be the flow volume from station $v_i$ to station $v_j$ in pick-up requests per time unit (e.g., unit loads per shift), for $i,k,1,2,\ldots,m$, and $w_i$ be the weight of pick-up requests associated with station $v_i$, for $i=1,2,\ldots,m$. The weight of pick-up requests associated with station $v_i$ can be computed as follows: $w_i = \frac{\sum_{j=1}^{m} f_{ij}}{\sum_{j=1}^{m} \sum_{j=1}^{m} f_{ij}}$, for $i=1,2,\ldots,m$. Since all stations have loads to transfer, $w_i > 0$, for $i=1,2,\ldots,m$ and, by definition, $\sum_{i=1}^{m} w_i = 1$.

The proposed MILP model uses the following decision variables:
Pazhani, Ventura, and Mendoza

\[ X_{ij} = \begin{cases} 
1, & \text{if station } v_i \text{ is assigned to AGV in dwell point } x_j, \\
0, & \text{otherwise}, 
\end{cases} \quad \text{for } i = 1, 2, \ldots, m, j = 1, 2, \ldots, p. \]

\[ \delta_j = \begin{cases} 
1, & \text{if an AGV is located in dwell point } x_j, \\
0, & \text{otherwise}, 
\end{cases} \quad \text{for } j = 1, 2, \ldots, p. \]

The assumptions considered in the model are given below:

a) We assume that \(|P| > n\).

b) The velocity of the vehicles is assumed to be constant.

c) Traffic interference between vehicles is not taken into consideration.

d) The loading time at the pick-up station, transport time to the drop-off station, unloading time at the drop-off station, and the time to return to the dwell point are insignificant.

e) It is assumed that an AGV can serve all the pick-up requests assigned to it.

f) All the pick-up requests from a station are served by the same AGV.

The following MILP model finds the minimum mean response time of the AGV system and the optimal set of dwell points:

\[(P) \quad \text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{p} w_i \left( \sum_{j'=1}^{p} r_{ij'} X_{ij} \right), \]

subject to

\[ \sum_{j=1}^{p} X_{ij} = 1, \quad \forall i = 1, 2, \ldots, m, \quad (1) \]

\[ \sum_{i=1}^{m} X_{ij} \leq |P| \delta_j, \quad \forall j = 1, 2, \ldots, p, \quad (2) \]

\[ \sum_{j=1}^{p} \delta_j = n, \quad (3) \]

\[ X_{ij} \in \{0,1\}, \quad \forall i = 1, 2, \ldots, m, \forall j = 1, 2, \ldots, p, \quad (4) \]

\[ \delta_j \in \{0,1\}, \quad \forall j = 1, 2, \ldots, p. \quad (5) \]

In this model, constraint set (1) ensures that each P/D station is assigned to a single AGV prepositioned at a precise dwell point. Constraint set (2) ensures that P/D stations are assigned to a potential dwell point \(x_j\) only when an AGV is assigned to \(x_j\) (\(\delta_j = 1\)). Constraint (3) ensures that exactly \(n\) dwell points are selected. Constraint sets (4) and (5) show the nature of variables considered in the model.

3. Genetic algorithm (GA) for locating dwell points in general guide-path layouts

Large-scale NP-hard problems are difficult to solve using mathematical models in a general computational sense. The time complexity of the problem increases exponentially as a function of the problem size. GAs are considered to be a powerful set of global search techniques that have been shown to produce very good results for a wide class of NP-hard problems ([9]). In this study, we propose a generic GA procedure for the dwell point location problem for general guide-path layouts. The GA procedure can be easily adopted to solve the problem with any linear/non-linear function of the response times by modifying the fitness function appropriately. The notations used in the procedure are listed below:

**Notation**

\( f_c \) : Fitness value of chromosome \( c \).

\( RP_c \) : Relative fitness of chromosome \( c \) in the parent population.

\( CP_p \) : Cumulative fitness of chromosome \( c \) in the parent population.

\( L \) : Length of the chromosome.
max_gen : Maximum number of generations (termination criteria).
MR : Mutation rate.
N : Population size.
o_gen : Number of the current generations.
par_pop : Chromosomes in the parent population.
off_pop : Chromosomes in the offspring population.
u : Uniform random number between 0 and 1.
Z(c) : Objective function value of chromosome c.

Chromosome representation
In this study, a feasible solution to the dwell point location problem can be represented in vector form. For a problem with p potential dwell points, the length of the chromosome (L) is equal to p. For example, consider a loop layout with 7 stations and 3 AGVs (i.e., 7 stations represent the potential dwell point locations). The permutation of potential dwell point indexes represents the chromosome. The chromosome representation is shown in Figure 2. In this example, the location of the dwell points would be in stations 1, 3, and 5.

<table>
<thead>
<tr>
<th>1</th>
<th>5</th>
<th>2</th>
<th>7</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
</table>

Figure 2. Chromosome representation

The step by step procedure for the proposed GA is presented in this section.

STEP 1: Initialize no_gen = 0.
STEP 2: Generate N chromosomes randomly for the initial population. Each gene represents a potential dwell point index and each chromosome represents a permutation of potential dwell point positions.
STEP 3: Evaluate the fitness function \( f_c \) of the chromosome in the initial population.
STEP 4: With the fitness value \( f_c \), calculate the selection probability \( RP_c \) and cumulative probability \( CP_c \) for every chromosome \( c \) in par_pop.
STEP 5: Perform crossover operation from par_pop, to obtain N offspring chromosomes. Set these offspring chromosomes to off_pop:
- In the GA-GX, off_pop is generated by using gene-wise crossover (GX) mechanism. In the GA-OX, off_pop is generated by using order crossover (OX) mechanism.
- In the GA-GX-OX, off_pop is generated using hybrid crossover mechanism. Generate u. If 0 ≤ u < 0.5, GX crossover is used to generate off_pop; if u > 0.5, OX crossover is used to generate off_pop.
- Detailed implementation steps of the GX and OX mechanisms are given in [8].
STEP 6: The N chromosomes in off_pop is subjected to mutation, with a probability of MR. We use the swap mutation operator in this study.
STEP 7: Evaluate the fitness function \( f_c \) of every chromosome in off_pop.
STEP 8: From both par_pop and off_pop, select the best N distinct chromosomes based on the fitness value, to form the par_pop for the next generation.
STEP 9: Increment no_gen = no_gen + 1;
- If no_gen < max_gen, then return to STEP 4; else proceed to STEP 10.
STEP 10: Stop. The best solution (i.e., the best chromosome) among the chromosomes in the final par_pop and its objective function value constitute the best known solution to the problem.

A detailed discussion on the fitness function, crossover mechanisms, and parameter settings of the GA are given in [8].

4. Illustrative example
In this section, we illustrate the application of the proposed MILP model and GA based methodology (with three types of crossover mechanisms) to a two-dimensional grid layout problem. We consider the objective of minimizing the mean response time. Consider the grid layout example in Figure 3. The number of stations (m) is 15. In this example, it is assumed that \( s_E = s_B = 1 \). There are seven intersections in the layout of which only two intersections (which are numbered as 16 and 17) have an out-degree 2. The other five intersections, with an out degree 1, do not
Pazhani, Ventura, and Mendoza

qualify as a potential dwell point (based on Theorem 1 in [8]). So, the number of potential dwell points \( p \) is 17 (15 P/D stations and 2 intersections). The number of AGVs \( n \) in the system is taken as 3. The distances between the stations and potential dwell points and the weight of pick-up requests associated with P/D stations are given in [8] (see Table 1 and Table 2).

![Image of two-dimensional grid layout](https://example.com/figure3.png)

**Figure 3.** Two-dimensional grid layout (illustrative example) [8]

<table>
<thead>
<tr>
<th>Table 1. Distance between the stations (illustrative example: two-dimensional grid layout)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From P/D station</td>
</tr>
<tr>
<td>1</td>
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<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Weight of pick-up requests associated with P/D stations (illustrative example: two-dimensional grid layout)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P/D station</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>4</td>
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<td>5</td>
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<td>6</td>
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<tr>
<td>7</td>
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<tr>
<td>8</td>
</tr>
</tbody>
</table>

We solve the illustrative example using the MILP model and the three versions of the GA for problem \( (P) \). The results are shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3. Results of the illustrative example: two-dimensional grid layout</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal solution (MILP) CPU time (MILP) in seconds</td>
</tr>
<tr>
<td>((P))</td>
</tr>
<tr>
<td>CPU objective GA objective</td>
</tr>
<tr>
<td>5.9839</td>
</tr>
<tr>
<td>Best GA objective</td>
</tr>
<tr>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>
GA is able to find the optimal solution for this illustrative example for problem \((P)\). The dwell point locations and the assignment of the stations to the dwell points are shown in Table 4.

### Table 4. Dwell point locations and their assignment (illustrative example: two-dimensional grid layout)

<table>
<thead>
<tr>
<th>((P))</th>
<th>Dwell point</th>
<th>Stations assigned to the dwell point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal and GA solution</td>
<td>Station 3</td>
<td>3, 4, 5, 6, 15</td>
</tr>
<tr>
<td>Station 13</td>
<td></td>
<td>1, 2, 8, 9, 13</td>
</tr>
<tr>
<td>Intersection16</td>
<td></td>
<td>7, 10, 11, 12, 14</td>
</tr>
</tbody>
</table>

The proposed GA procedure yields an optimal solution for the two-dimensional grid layout problem. A computational study has also been performed on loop layout and two cases of two-dimensional grid networks in [8] and the results are promising. The average deviation of GA solution from optimal is 0.88% and less than 2% for loop layout and two-dimensional grid network problems respectively ([8]).

### 5. Conclusions

AGVs provide a promising solution which can improve productivity of these systems along with reducing labor costs, material handling damage, and increasing dependability and safety. General directed guide-path layouts, such as grid type layouts, are becoming more common in manufacturing and warehouse systems for efficient usage of storage space and to reduce travel times between storage locations. In this article, we have considered the problem of locating dwell points for idle vehicles in an AGV system. We have addressed the problem of optimally determining dwell points for multiple AGVs in general directed networks with the objective of minimizing the mean response time. We have developed a MILP model and a generic GA procedure for solving the problem. We have illustrated the model and the GA procedure using a two-dimensional grid layout example. The proposed GA was able to give optimal solution for the illustrative example. Our computational study on the loop layout and two-dimensional grid network problems show that the GA procedure was able to find near optimal solutions in reasonable computation times. This proves the potential of the proposed GA to handle real-time problems.

### References


