A SHIP Inventory Routing Problem with Heterogeneous Vehicles under Order-Up-To Level Policies

Saijun Shao*, George Q. Huang1
1HKU-ZIRI Lab for Physical Internet, Department of Industrial and Manufacturing Systems Engineering, The University of Hong Kong,
Pokfulam Road Hong Kong, PR China
*Email: aaron.shaosj@gmail.com

Abstract
This paper studies a variant of Inventory Routing Problem (IRP) in the context of Supply Hub in Industrial Park (SHIP), where a cluster of closely located manufacturers share the warehousing and transportation services provided by the Supply Hub. We aim to identify a combined inventory and routing strategy to minimize the total cost of inventory holding and transportation services over the finite planning horizon. In this paper, the SHIP-IRP is formally described and modeled as a two-level, multi-period, one-to-many distribution system. Heterogeneous vehicles have been incorporated into IRPs for the first time, and three assumptions have been made to make the model computationally solvable.

Keywords
Supply Hub in Industrial Park (SHIP), Inventory Routing Problem (IRP), Heterogeneous vehicles, Order-up-to level policy

1. Introduction

This paper studies a distribution system which consists of a single public warehouse and a set of geographically dispersed retailers. Replenishments are motivated by retailers’ deterministic demands, and carried out by a fleet of heterogeneous capacitated vehicles. We aim to identify a combined inventory and routing strategy to minimize the total cost of inventory holding and transportation services over the finite planning horizon. This insight was inspired by the context of Supply Hub in Industrial Park (SHIP), in which a cluster of manufacturers locating nearby to each other share common resources. The Supply Hub acts both as a public warehouse and a transportation service provider for the manufacturers.

In our model, we consider a two-level inventory system: the majority of goods are kept at the central depot, and certain amount of stock (usually small quantity) is held at manufacturers’ plants. This framework is motivated by the following reasons: 1) Orders of raw materials are usually large so that enterprises can benefit from economies of scale, however this could conflict with the space limitation and also incur high warehousing costs. What’s more, variations of stocks might occur because of customers’ fluctuating needs and seasonal effects for some manufacturers, which will result either in high stock-out possibility or expensive inventory holding cost. To address this problem, Qiu et al. (2011) study on the storage capacity pooling effect, and have proved that pooling inventory from multiple separated warehouses into a central depot will mitigate the inventory variations, and significantly reduce the stock holding cost benefiting from sharing of storage space, facilities and management. 2) On the other hand, manufacturers need to keep a certain level of stock, or called buffer, so as to ensure smooth, continuous production. Therefore certain amount of inventory shall also be held at manufacturers’ plants, and the restocking of buffer is carried out by a fleet of shared vehicles through shipping goods from the central depot to manufacturers according to their respective consumption rate. Thus in the SHIP inventory routing problem, decisions need to be made upon coordinating the transportation and the two-level inventory system.

* Corresponding author
We now introduce in detail some properties of our SHIP-IRP, which are very different from those of general problems studied in existing IRP literatures.

Firstly, as mentioned above, we have two levels of inventory, namely the public warehouse and the buffer stock at plants. They are of same importance whereas they possess very different properties. For the central warehouse, the capacity is assumed to be unlimited, and stock holding cost need to be charged because it’s significant. Buffer stocks, however, are assumed to be free of charge because the amount is relatively small; instead, it’s the level of inventory at any time that we are concerned about. We require the buffer level at any time to be maintained between a maximum and a minimum level which are predetermined by customers. This quasi-JIT mechanism will generate shipments of small quantity and high frequency, the Supply Hub thus needs to combine deliveries into efficient routes so as to minimize the total cost while meeting all manufacturers’ demands.

Secondly, there exist peak and off-peak hours of material consuming rates during a single day in the Industrial Park according to reality. For instance, at the early beginning of a day, only a few manufacturers start production, and this could be an off-peak period. While later on when most manufacturers start production, larger amount and more frequent replenishments will be demanded, and this is called the peak period. The non-uniformly distributed demands will lead to the need of utilizing vehicles with different sizes, i.e. “big” vehicles shall be used at peak hours to cut costs through economies of scale, while “small” ones to be used at off-peak hours to enhance the utility of vehicles. Our research can also be easily extended to cope with the problem where some special products such as bulky or dangerous items need to be handled by special vehicles.

Thirdly, unlike inter-city cases where customers are located far away from each other, we study the inner-city case of Industrial Park, where manufacturers are situated relatively close to each other. Hence the distance between any two nodes would not be significantly different and is assumed to be constant in our work. Consequently the travelling cost and time between any two nodes would be the same given the same type of vehicle being used. The routing problem is then reduced to a clustering problem, in which we only need to divide customers which need to be serviced within the same period into several clusters and assign to each cluster a certain type of vehicle, because the sequence does not need to be considered given the identical distance between any two nodes. This assumption makes our model different from most of the vehicle routing problems, whereas we deem it reasonable under our specific circumstance. This assumption can also be compensation to the increased computational complexity caused by the incorporation of heterogeneous vehicles.

2. Literature review

The concept of Supply Hub in Industrial Park, SHIP, initially put forward by Qiu et al. (2010), is referred to as applying supply hub to an industrial park to cope with problems occurring during its further development, especially the shortage of land resources. Remarkable values of public warehousing and transportation sharing have been demonstrated in their following studies, respectively (Qiu and Huang, 2011; Qiu and Huang, 2013). Inventory and vehicles routing problems are traditionally studied separately, however the integration decision of these two problems, known as Inventory Routing Problems (IRPs), is believed to dramatically enhance the system performance. Past studies have proved the significant overall cost reduction through simultaneously identifying optimal inventory and transportation strategies, comparing with results of solving these two problems separately (Golden et al., 1984; Bell et al., 1983; Federgruen and Zipkin, 1984; Dror and Ball, 1987). This paper thus aims to extend the study of SHIP in the sense of integrating inventory and transportation problems so as to further reduce the system-wise costs.

Golden et al. (1984) are amongst the first ones to investigate the Inventory Routing Problem, which is a variant of the elementary Vehicle Routing Problem (VRP). In typical IRPs, inventory and transportation costs are coordinated through simultaneously determining the replenishment policies and the vehicle routing so as to minimize the total cost of logistics and stock holding. Most researches model the inventory routing problem as a one-to-many distribution system, in which a single supplier is responsible for restocking a group of customers. Studies into IRPs can be broadly categorized in terms of (finite or infinite) planning horizon, (single or multiple) period, and (deterministic or stochastic) demand (Moin et al., 2011). To deal with practical problems in SHIP, we focus on the finite, multi-period, deterministic version in this paper.

VRPs have been notoriously hard to solve, the incorporation of inventory cost even adds to the computational complexity. Thus, almost all researches of IRPs study the cases in which replenishments are carried out with identical
vehicles (Federgruen and Zipkin, 1984; Bertazzi et al., 2002; Chan et al. 1998; Bard et al. 1998; Moin et al. 2011; Campbell et al. 2004; Anily and Federgruen 1990; Jaillet et al. 2002). Although heterogeneous vehicles have been considered and studied in some variants of VRPs, we believe in the context of IRPs the mutual impacts between warehousing cost and vehicle selection deserve further research. To the best of our knowledge, this paper is the first attempt to incorporate the use of heterogeneous vehicles in an Inventory Routing Problem.

The contribution of this paper is threefold. Firstly, we formally describe and model the SHIP inventory routing problem, which is a promising variant of IRPs yet distinctly differs from general models studied in existing literatures. Secondly, this paper is the first attempt to consider the use of mixed vehicles in an Inventory Routing Problem. We incorporate vehicles with different sizes into our model and make it computationally solvable through several amendments to the traditional IRP model. This could also shed light on dealing with the problem where special goods such as bulky or dangerous items need to be handled with special vehicles. Thirdly, this research will contribute to the development of business mode of SHIP, namely the mechanism of designing the warehousing and transportation services.

3. Problem description

Our problem can be stated as follows: a set of customers share a central depot, denoted as 0, to keep their inventory, and replenish their buffer stocks at hand by shipping goods from the depot. Each customer absorbs a single, unique type of product. The central depot is responsible for placing orders for customers to outside suppliers with constant, product specified, lot sizes. Moreover, we assume that the lot size for each product is reasonably larger than the amount consumed by the customer within each single period, so that the order placed to outside suppliers would be exactly the amount of the corresponding lot size. In each discrete time period, a deterministically known amount of products are consumed at customers, and shipments are carried out to keep the buffer stock level at customers to be between the maximum level and the minimum level, which are predetermined by customers. The supply hub operates a fleet of heterogeneous vehicles, and the number for each type in unlimited in our problem. The capacities of different types are varied, and shared by all kinds of products. Vehicles set off from depot at the beginning of each period, finish shipments and return to the depot before this period ends. Minimum cost inventory and routing strategies need to be identified while meeting all customers’ requirements.

In consideration of computational feasibility, especially concerning the incorporation of heterogeneous vehicles, we have come up with three assumptions to simplify the model.

1) The inventory at customer is assumed to be replenished under an order-up-to level policy, in which the quantity to be delivered is equal to the amount that would enable the customer’s inventory level to reach its maximum capacity after each shipment. Similar assumption can be found in Bertazzi et al. (2002). This policy helps us to eliminate a set of continuous decision variables from the model, in the way that we can simply calculate the quantity to be delivered based on other decision variables, instead of identifying the optimal replenishment amounts. This will not remarkably harm the effectiveness, because in our problem the buffer level is relatively low, the delivery amount obtained indirectly will be reasonably close to the optimal result. This assumption, however, will significantly reduce the computational complexity due to the removal of a set of continuous variables.

2) The distance between any two nodes of the distribution system, including the depot and manufacturers, is assumed to be a known constant. This assumption stems from the observation that customers and the central depot inside an Industrial Park locate close to each other, and the travel time (cost) between any two nodes are not significantly different. Traditional vehicle routing problems consider the sequence of customers to be served within the same route. In our model, however, we divide our time horizon into many smaller time periods, and it will not matter a lot whether a customer is served at the beginning or at the end of the same period. The single vehicle hence does not need to consider the order to serve the assigned customers as long as it can finish all the shipments within the current time period. This assumption also greatly cuts the computational complexity.

3) We assume that each manufacturer absorbs a single, unique type of item. This assumption will not harm the flexibility because a manufacturer absorbing more than one type of item, say C types, can be divided into and treated as C different manufacturers of which each absorbing one item, setting the travel cost between the artificial
manufacturers of the same set to be zero, and setting travel cost between artificial manufacturers from different sets same as the travel cost of the two original nodes. This idea is also illustrated in Bertazzi et al. (2002). With this consideration, each type of product is designated for its unique, corresponding customer. Nevertheless, this assumption also indicates that the total amount of customers might be very large in practical cases, resulting in great computational complexity. Exact algorithms are thus believed to be inefficient, if not impossible, to tackle this problem.

We hereby summarize our notations as follows:

Indices:

- \( m \) : denotes customers, \( m \in \mathcal{M} = \{1, 2, \ldots, M\} \)
- \( i, j \) : denote depot and customer locations, \( i, j \in \mathcal{M}' = \mathcal{M} \cup \{0\} \)
- \( t \) : denotes a discrete time period, \( t \in \mathcal{T} = \{1, 2, \ldots, T\} \)
- \( k \) : denotes vehicle type, \( k \in \mathcal{K} = \{1, 2, \ldots, K\} \)

Input Parameters:

- \( M \) : number of customers
- \( T \) : number of time periods
- \( K \) : number of types of vehicles
- \( P_i \) : type of product consumed by customer \( i, i \in \mathcal{M} \)
- \( h \) : unit inventory holding cost per time period at the depot
- \( LS_i \) : lot size of \( P_i \) when depot places orders to outside suppliers
- \( VC_k \) : vehicle capacity of type \( k, k \in \mathcal{K} \)
- \( LI_i \) : minimum inventory level of customer \( i, i \in \mathcal{M} \)
- \( HI_i \) : maximum inventory level of customer \( i, i \in \mathcal{M} \)
- \( f_k \) : fixed cost of each route with vehicle type \( k, k \in \mathcal{K} \)
- \( v_k \) : unit distance transportation cost with vehicle type \( k, k \in \mathcal{K} \)
- \( R \) : constant distance between any two nodes in the network
- \( \tau \) : constant travel time between any two nodes in the network
- \( t_0 \) : constant time span of each time period
- \( D^t_i \) : quantity of \( P_i \) consumed by customer \( i \) during time period \( t, i \in \mathcal{M}, t \in \mathcal{T} \)
- \( I^t_i \) : initial inventory level of \( P_i \) at the depot, \( i \in \mathcal{M} \)
- \( CI^t_i \) : initial inventory level of \( P_i \) at customer \( i, i \in \mathcal{M} \)

Decision Variables:

- \( X_{ij}^t \) : 1, if \( P_i \) with volume \( LS_i \) is made available at the depot during time period \( t, i \in \mathcal{M}, t \in \mathcal{T} \)
  0, otherwise
- \( Y_{ijk}^t \) : 1, if arc \((i, j)\) is traversed by a type \( k \) vehicle during time period \( t, i, j \in \mathcal{M}', t \in \mathcal{T}, k \in \mathcal{K} \)
  0, otherwise

In order to present our model more clearly, we also introduce the following notations:

- \( I^t_i \) : inventory level of \( P_i \) of the depot at the end of time period \( t \)
- \( CI^t_i \) : inventory level of \( P_i \) at customer \( i \) at the end of time period \( t \)
- \( S^t_i \) : quantity of \( P_i \) to be delivered to customer \( i \) during time period \( t \)
- \( A^t_i \) : arrival time of vehicle of type \( k \) at node \( i \) during time period \( t \)
- \( \psi_{ij}^t \) : total load of a type \( k \) vehicle on the arc \((i, j)\) during time period \( t \)

The problem is then formulated as follows:
Minimize  
\[ \sum_{i \in \mathcal{M}} \sum_{k \in \mathcal{K}} h t_{ik}^t + \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{M}} Y_{jk}^t + R \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{M}} \sum_{i \in \mathcal{M}} Y_{ijk}^t \]  

(1)

in which

\[ t_{ik}^t = t_{ik}^{t-1} + X_{ik}^t \cdot LS_t - S_{ik}^t, \quad \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(2)

where

\[ S_{ik}^t = \begin{cases} H t_{ik} + D_t - c t_{ik}^{t-1}, & \text{if } \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{M}} Y_{ijk}^t = l, \\ 0, & \text{otherwise} \end{cases} \quad \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(3)

\[ C t_{ik}^t = c t_{ik}^{t-1} + S_{ik}^t - D_t, \quad \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(4)

Subject to

\[ t_{ik}^t \geq 0, \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(5)

\[ C t_{ik}^t \geq L t_{ik}, \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(6)

\[ \psi_{ik}^t = S_{ik}^t \cdot Y_{ik}^t + \sum_{k \in \mathcal{K}} \psi_{ik}^t, \forall \ j \in \mathcal{M}, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \]  

(7)

\[ \sum_{i \in \mathcal{M}} Y_{ik}^t = \sum_{j \in \mathcal{M}} Y_{ij}^t, \forall \ j \in \mathcal{M}, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \]  

(8)

\[ \sum_{i \in \mathcal{M}} \sum_{k \in \mathcal{K}} Y_{ijk}^t \leq 1, \forall \ j \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(9)

\[ 0 \leq \psi_{ik}^t \leq V C_k, \forall \ i \neq j \in \mathcal{M}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \]  

(10)

\[ A_{ik}^t = \sum_{j \in \mathcal{M}} (A_{ik}^t + \tau) \cdot Y_{ijk}^t, \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \]  

(11)

\[ A_{ik}^t = (t - 1) \cdot t_0, \forall t \in \mathcal{T}, \forall k \in \mathcal{K} \]  

(12)

\[ A_{ik}^t \leq t \cdot t_0 - \tau, \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(13)

\[ \psi_{ik}^t = \psi_{ik}^t = 0, \forall \ i \in \mathcal{M}, \forall k \in \mathcal{K} \]  

(14)

\[ Y_{ijk}^t = 0 \ or \ 1, \forall \ i, j \in \mathcal{M}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \]  

(15)

\[ X_{ik}^t = 0 \ or \ 1, \forall \ i \in \mathcal{M}, \forall t \in \mathcal{T} \]  

(16)

In the above formulation, (1) is the objective function which refers to minimizing the overall cost. The first part represents the total inventory holding cost at the central depot over the whole time horizon. The second part relates to the fixed cost of routes, and the third part is the variable transportation cost.
Equations (2)-(4) illustrate how to calculate the inventory level at the central depot and at customers’ plants. To be more specific, equation (2) reveals that the inventory level of a certain type of product at the end of the current period is derived by the level at the end of its previous period, plus the amount made available during this period (if any), minus the amount shipped to the corresponding customer (if any). Equation (3) shows that under the order-up-to level policy, if a shipment is executed to a customer, the delivered amount is determined by the maximum inventory level plus the amount consumed during the current period, minus the buffer stock level at the end of its previous period. Equation (4) illustrates the buffer level at a customer can be calculated as its inventory level at the end of its previous period plus the amount replenished, minus the amount absorbed during the current period. Actually (2) and (4) can be viewed as the links between successive discrete periods in our problem, and show how the decisions of past periods can impact the consequent periods.

Constraints (5) ensure that the inventory level of any kind of product is non-negative at the depot. Constraints (6) specify that the buffer level should exceed the minimum requirements. Note that under the order-up-to level policy, the customers’ buffer level would never exceed the maximum level, thus this requirement has been eliminated from the constraint inequalities. (7), (8) and (9) together make sure a vehicle delivers exactly the amount a customer needs according to the order-up-to level policy when it visits the customer, and maintains the rest (if any) on the vehicle after it leaves current customer. Constraints (9) ensure that each customer will be visited at most once during each time period. Constraint (10) is used to keep the total load on the vehicle non-negative and within its capacity all the time. (11)-(13) force the vehicle to finish all the replenishments and return to the depot before the end of the current time period. Finally, (15) and (16) indicate that the decision variables are all binary.

4. Conclusion

In this paper, the SHIP-IRP is formally described and modeled as a two-level, multi-period, one-to-many distribution system. Inventory holding cost at the central warehouse, transportation cost and the capacity constraint at manufacturers’ plants have been coordinated to simultaneously identify optimal inventory and logistics decisions. For the first time, heterogeneous vehicles are incorporated into an Inventory Routing Problem to deal with a case where peak and off-peak hours exist. In order to make it computationally solvable, we consider three methods to simplify our model, and demonstrate their rationality in the context of SHIP.

Future researches could be conducted on several aspects. Firstly, approaches need to be sought among the existing algorithms to solve the proposed model efficiently and effectively, with adjustments bound to be made due to the assumptions that we adopt in our model. Secondly, numerical studies will enable the analyses of how the parameters, for instance the distribution of demands, the capacity of various types of vehicles, the inventory constraint at customers etc., would influence the decision and the performance of the system. Thirdly, to approximate the realities, some restrictions in our model could be relaxed, for instance, a customer can be visited more than once during a single period, or consider uncertainties of transportation and consumption rate of raw materials. The result of this paper could provide some insights into the development of operation mode of SHIP, and act as a benchmark to compare with alternative decision mechanisms.

References

7. CHIOU S W. Integrating the inventory management and vehicle routing problems for congested urban 
(SHIP): The impact of demand uncertainty, Proceeding of the IEEE International Conference on Industrial 
Computers & Industrial Engineering 65, 16-27.