Under-Water Sonar Placement

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Abstract
Given a marine area of interest (e.g., water-ways, ports), this paper attempts to efficiently allocate sonar sensors (utilized for under water detection) within a limited budget to achieve a reasonable amount of coverage, thereby keeping ports and waterways under surveillance against impending threats. Though total coverage of the entire area of interest is desired, priority is given to the criticality/importance attached to specific locations within the area of interest, on a grid-based system. We present two optimization models to detect potential threats by placing underwater sonars within a port or waterway of interest. The first model (mixed integer linear programming model) developed maximizes the detection coverage based on the importance attached to a sub-region. The second model is a mixed integer nonlinear programming model that is designed to achieve optimal detection based on a desired detection probability threshold. Both models satisfy a budget constraint that limits number of sonars to be deployed.

Keywords
Sonar Placement, Optimization, Grid System, Marine.

1. Introduction
It is widely accepted that over 90% of world international trade travels by sea. As such, global economic interdependency among cities and nations is largely dependent on the success of the maritime industry. With the incidence of terrorism and arson attacks, it becomes expedient to protect ports, waterways and other maritime infrastructures from these attacks.

This need for protection becomes increasingly important due to the ever-growing maritime industry. As a response to this growth in the maritime sector, the US Department of Homeland Security (DHS) developed the National Strategy for Maritime Security (NSMS) of 2004 (DHS 2004) that has the objectives to enhance international cooperation, to prevent terrorist or criminal attacks in the maritime domain, to protect maritime-related population centers and critical infrastructure, to minimize damage and expedite recovery following an incident in the maritime domain, and to safeguard the ocean and marine resources [1].

Many methods of detecting the presence of underwater targets have been investigated. Some of the non-acoustic methods include use of Magnetic signatures, Optical signatures, Electric field signatures, Thermal detection (infrared), Hydrodynamic changes (pressure), etc. However, none of these methods is known to be as effective as acoustic sonars. The deployment of acoustic sensors serves as a viable alternative in the detection of these water-borne threats. Amongst other advantages, the salinity of water bodies does not affect their performance. Hence their use in underwater threat detection is justified.

Most surveillance systems, especially wireless sensor networks deployed in most applications, detect objects by monitoring an environment and consequently sending feedbacks to a supervising control unit before necessary actions are initiated. The focus of this paper is solely restricted to this in-field monitoring and excludes any operation taking place upstream in the supervising control unit.

Underwater threat detection is widely applicable in military and civil applications, often dealing with protection of critical assets. In the maritime environment, sustained surveillance of maritime infrastructures and resources such as
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harbors, high-risk sea routes and ports is a critical requirement of ensuring the safety and security. Incidents of piracy, terrorism and illegal activities such as oil bunkering are security threats that need to be addressed in this domain. Among other issues relevant to the design of an appropriate surveillance system using sensors, sensor-deployment remains a subject of numerous researches. Sensor deployment is an important area of research due to the difficulty of determining optimal coverage with a limited amount of sensors. The complexity of determining optimal sensor placement is caused by the properties of the environment, the types of obstacles in the environment, and the sensing phenomena [2]. We exclude false alarm probabilities in our model formulation. In response to the observation that the incidence of false alarms in a surveillance region such as marine environment involving deployment of torpedoes remains critical [3], we make reference to a fact earlier expressed that most on-field surveillance systems send feedbacks to a supervising control unit before evasive or interceptive actions are taken. In addition, like [4], we make the assumption that a perfect communication exists between on-field surveillance and supervising control unit.

Sensor Deployment: Grid-based placement

A sensor network can be deployed in two ways – with random placement or with grid-based placement. When the environment is unknown, random placement is the only choice and sensors may be thrown to any place by aircrafts randomly. The alternative is to deploy sensors on a sensor field, if the properties of the terrain are predetermined. The field is generally divided into grids and sensors are carefully deployed at the grid points. This approach is called grid-based placement [5]. Dividing a region of interest into cells on a grid basis enables depiction of the significance/importance of each cell by parameter values. Thus, sensor coverage for a cell is prioritized based on the cell parameter values.

Unlike most sensor network problems, especially in the wireless network domain which often deals with full coverage (i.e. all grids are required to be covered by at least a sensor); underwater sonar deployment usually deals with partial coverage with preference given to sub regions within the network grid considered to be more important in comparison with other regions.

The most common grid used in sensor deployment is the conventional square (rectangular) grid network. However, it is known from literature that a triangular coordinate system (triangular grid or its derivatives such as the hexagonal grids) in sensor placement will offer a more efficient way of area coverage if a sonar is to be placed in each component cell. By extension, sensor placement in a region/cell given the capability of the sensor to cover its cell of placement and nearby cells will be more efficient when the chosen coordinate system is triangular.

The optimal tessellation using regular triangles with side length equal to $\sqrt{3}R_s$ achieves the minimum sensor density for complete area coverage, where ‘$R_s$’ is the radius of the sensor sensing disk [6]. Indeed, other regular polygons can also be used in sensor grid deployment.

Our Contributions

Our study contributes to the literature of maritime security risk management and mitigation by proposing mathematical models for placing underwater sonars in a maritime environment to monitor potential underwater threats. Unlike terrestrial sensors, sonars are quite expensive. Thus, this placement methodology is to be achieved under budget limitations.

As [7] rightly observes, “The key benefits of terrestrial sensor networks stem from wireless operation, self-configuration, and maximizing the utility of any energy consumed. They emphasize low cost nodes (around US$100), dense deployments (at most a few 100m apart), short-range, multi-hop communication; by comparison, underwater acoustic communication today are typically expensive (US$10k or more), sparsely deployed (a few nodes, placed kilometers apart), typically communicating directly to a “base station” over long ranges rather than with each other.” Within the context of risk analysis, our sonar placement problem is formulated thus: Given a limited amount of budget and a marine area of interest, the sonar placement problem is to efficiently determine the number and regional locations to allocate different sonar types of different sonar coverage orientations and ranges such that detection probabilities is optimized based on the criticality (regional importance) attached to these regions. The rest of this study is presented in the following sequence: Section 2 provides an extensive literature review/survey of general sensor placement problem without restriction to the maritime security domain. Particularly, related research in general wireless communications and remote monitoring are included. The approach we have adopted, along with the mathematical models are provided in section 3. While section 4 details an example deployment in a maritime environment, section 5 presents results obtained. Though hypothetical, the example case study which involves sonar deployment in a sea route network traversed by ocean-going vessels shows the applicability of the methodology. Section 6 provides conclusion and our current on-going works related to the placement methodology.
2. Literature Review

Without restricting the literature on sensor placement solely to the maritime sector, there exist vast amounts of literature directly or indirectly related to the subject matter. Irrespective of the problem domain or problem statement, these works involve attaining an efficient placement methodology to satisfy a given requirement (or requirements) in the presence of some inhibiting factors (ranging from connectivity, physical structures, economic considerations, etc.).

According to [1], many of the citations in literature use a more quantitative and formal approach to evaluate and mitigate risk and calculate the expected consequences based on the types of possible incidents thus:

\[ E(C) = \sum_j E(C|I_j)p(I_j) \]

where \( E(C) \) and \( E(C|I_j) \) are the expected consequence and the expected consequence given an incident type \( j \) respectively; and \( p(I_j) \) is probability that incident \( j \) occurs. However, not all attacks lead to an incident. Hence, the risk literature further highlights dependence of \( p(I_j) \) on threat probabilities, \( p(E_a) \), asset vulnerability and expected consequence in the presence of an attack. Kindly see [8] and [1] for more elaborations.

Approach undertaken by works in this respect is divergent. While some existing works in literature are able to individually include all these components of \( E(C) \), others concentrate on either 1 or 2 of them [1]. For this paper, we concentrate on considering \( E(C) \) as a composite and do not individually consider each of its dependencies in our formulations.

Ghafouri and Altiok [1] provides a compendium of works related to the defense of critical infrastructures and argued that the papers were in response to security challenges as a result of terrorist attacks, especially after the 9/11 incident. Some of these works include: [9], [10], [11], [12], [13], [14], [15], [16], etc.

A generic approach in protecting critical infrastructures is to improve their surveillance by installing detective instruments such that the probability of undetected attack and its success decreases. Sensors are the most widely known detective objects used in this field and the classical Sensor Placement Problem (SPP) in optimization theory is used in deployment strategies. Based on the specific placement problem being considered, SPP is usually addressed under the following distinct problem areas: **Point Coverage, Barrier Coverage and Area Coverage**. Point Coverage essentially deals with providing sensor coverage to specific infrastructures; Barrier coverage is involved with ensuring sensor coverage is provided to routes leading to an infrastructure and Area Coverage deals with providing same to an entire region. While earlier works broadly considers the SPP in general terms, latter works are often dedicated to these distinct problems.

Sensors have been successfully used in many applications related to surveillance operations. Meguerdichian et al [17] discusses the problem of determining the coverage provided by a given placement of sensors. Chakrabarty et al [18] and Dhillon et al [19] provide models for grid coverage sensor placement to keep a region under surveillance and use heuristic techniques to solve their models. Clouqueur et al [20] deploy sensors on the ground using a sequential heuristic technique such that the exposure of the paths (leading to the infrastructures) to sensors is maximized and hence the surveillance is maximized.

Shakottai et al [21] investigate the dependence of detection capabilities of a sensor network on sensor-density. Dhillon and Chakrabarty [22] present two algorithms for the efficient placement of sensors in a sensor field to optimize the number of sensors in a distributed sensor network. To take care of imprecise detections and terrain properties, a probabilistic approach is adopted in the algorithms’ development. The approach is demonstrated using experimental results from an example sensor field with inherent obstacles.

Nagy [23] builds a family of n-planes triangular grids: **Hexagonal, Triangular and 3-planes triangular grids** and shows geometric interpretation of both the hexagonal and triangular grids: Both can be considered as sets of points in \( \mathbb{Z}^3 \). It considers hexagonal and triangular grids as 1st and 2nd members of a family of triangular grids (n-plane triangular grids); and describes the neighborhood structure of the duals of the n-plane graphs. Nagy [24] presents suitable method for formulation of neighborhood sequences in a triangular grid.

Nagy [25] analyzes the 3-plane triangular grid and its dual from the basis of their neighboring conditions; and thereafter extends this analysis to n-planes grid. Other grid systems such as ‘circular’ three-plane grid and the higher dimensional triangular grids are also analyzed. Nagy and Strand [26] mainly deals with 3D graphs. Amongst others, they show how non-standard 3D models can be embedded in \( \mathbb{Z}^2 \) and how a triangular grid can be described using \( \mathbb{Z}^3 \).

Lifetime of a sensor network in terms of energy limitations are presented in [27]. Esseghir et al. [28] investigates the relationship between network lifetime and sensor coverage, maximizing the network lifetime. Lin and Chiu [5] develop a model for deployment of a set of sensors on a grid point to monitor the sensor field under the constraints of cost limitations to achieve complete coverage. Coverage is considered to be ‘full’ if distance between the particular
grid point and a sensor is less than the sensor’s detection radius. Otherwise coverage is assumed to be ineffective (binary decision). The model formulated minimizes the maximum distance between grid points and sensor locations and is formulated as a combinatorial optimization problem. An algorithm based on simulated annealing is also presented for large scale problems.

Ngatchou et al. [29] present computationally efficient greedy algorithms for optimal placement of multi-static sonar sensor placements with cost and coverage as principal objectives using particle swarm optimization. Heidemann et al. [7] summarize their then ongoing research in underwater sensor networks, highlighting potential applications and research challenges. Schoelhammer et al. [2] evaluates two different approaches to sensor placement, combining spline-based modeling, principal component analysis, data partitioning and Spatial-temporal statistical analysis. They develop an ILP formulation to the network deployment problem.

Kraus et al. [30] uses Gaussian Processes (GPs), a non-parametric probabilistic model for the spatial phenomena of interest and for the spatial variability of link qualities, taking into consideration estimation of predictive power and communication cost of un-sensed locations. They also developed an algorithm which selects the Sensor Placements at “informative” and “cost-Effective” locations. Armaou et al. [31] uses linear parabolic partial differential equations to study optimal sensor placement in the presence of disturbances.

Ibrahim et al. [32] present the underwater placement problem as an Integer Linear Programming and analyzed the tradeoff between the number of surface gateways, the expected delay and energy consumption. Altnel et al. [33] also presented formulations similar to [18] to model the least number of sensors required to cover interested targets of interest. Pashko et al. [34] develop an optimization framework for optimal placement of sensors within an urban harbor for underwater surveillance and detection of underwater threats. They develop single-period and multi-period models for optimal sensor placement and include statistical algorithms for threat detection and corresponding optimization problems for sensor placement. Numerical experiments they conducted demonstrate that the developed algorithm consistently outperforms [22]’s efforts. Lee and Kulesz [35] propose a sensor placement methodology to protect the population against exposure to chemical, biological and radiological threats.

Li et al. [36] studied the constrained surface-level gateway placement (C-SGP) problems in Underwater Acoustic Wireless Sensor Networks (UA-WSNs) with a focus of addressing the connected and survivable C-SGP problems to meet the connectivity and survivability requirements for some application environments of UA-WSNs. Two approximate algorithms are presented and their performance is justified with theoretical analyses. Wilhelm and Gokce [37] provide a model to design a surveillance system for ports and waterways by utilizing various types of sonars to detect anomalies above water. They propose a mixed integer linear model and use the branch and price decomposition technique to solve it. Golen et al. [15] provide a sensor allocation scheme (based on game theory) to protect a specific part of the ocean against submarine threats. Castello et al. [38] presents a novel strategy in determining an optimal sensor placement scheme for environmental monitoring using Wireless Sensor Networks (WSN) and accomplishes this by minimizing the variance of spatial analysis based on randomly chosen points representing the sensor locations. A simple example of measuring mercury in soil is illustrated in finding the optimal sensor placement using WSNs.

Dimitrov et al. [39] propose a mixed integer linear optimization method to deploy radiation detectors against nuclear material smugglers in a transportation network. Of the two specific aspects considered by [40] in their work on the protection of regional infrastructure from covert attack, its first aspect which deals with placement of sensors for detection of vehicles on the transportation network that pose a potential threat to regional infrastructure is most relevant to our work. Their methodology and formulation addressed an infrastructure’s vulnerability and attempted to decrease attack probabilities along potential attack routes.

Ghafoori and Altiok [1] propose an optimization model to keep ports and waterways under surveillance against threats. The proposed mathematical model incorporates the feature of multiple sonar coverage and range-dependent detection probability. In addition, a greedy heuristic approach which attains near-optimal solutions for large scale scenarios is presented.

Piracy, terrorist attacks and arson attacks have evolved to targeting areas of interest rather than specific targets in the hope of having better odds of accomplishing their heinous crimes. Therefore, out of the 3 identified SPP problems, we address the area coverage problem within a maritime environment.

The coverage problem entails a sonar is able to provide coverage to its region of placement as well as surrounding regions within the limits of its coverage. However, degradation exists in these detection probabilities as we recede away from the point of sonar placement. This degradation is known to be non-linear in nature. Some works have modeled this gradual degradation in sonar strength/sensor strength using exponential functions with parameters determined from the specific sensor characteristics. While these parameters are not only difficult to estimate, the approach also increases the complexity of the problems, introducing non-linearity to model multiple-detection based on sonar range.
As such, our methodology entails a division of a sonar influence into 3 distinct regions: Primary (Point of sonar placement), Secondary and Tertiary regions. We are of the opinion that rather than assigning a function to the detection probabilities in these regions, experts’ opinion should be solicited.

3. Model Formulations
We present two different models to achieve optimal sonar placements. While both models maximize sonar coverage (minimizing non-coverage) within a limited budget, the first model (Model #1) identifies regions with higher criticality and preferentially allots sonars to these regions. On the other hand, the second model (Model #2) attempts to maximize the total number of regions with detection probabilities above a certain threshold detection probability.

3.1 Model Parameters
- \( h_{ik} \) = Criticality index: Used to reflect relative importance assigned to a section/portion of the AOI (Area of Interest). Expert judgments and aid of a utility function may be used in this regards.
- \( a \): Index for types of static sonars.
- \( p_{az} \) = Probability of detection for static sonar type \( a \);
- \( z \): Index for static sonar’s coverage strength, ranging from 0 to a small integer;
- \( z \epsilon \{p\text{ (primary coverage)}, q \text{ (secondary coverage)}, t \text{ (tertiary coverage)}\} \)
- \( c_a \): Unit cost of static sonar.
- \( \text{cover}_{\text{max}} \): Maximum number of coverage a cell can have.

Set:
- \( N_{ikaz} \)=Set of cells in the neighborhood of cell, \( ik \) (which has a static sonar of type \( a \) placed in it) that can be covered by cell \( ik \) with detection probability \( p_{az} \);
- \( \gamma \) = Probability detection Threshold.
- \( R_{ik} \) = Integer variable introduced to identify cells \( ik \) whose probability exceed the threshold, \( \gamma \).

3.2 Decision Variables
- \( X_{ik}^a \) = \begin{cases} 1 & \text{If static Sonar of type } a = 1,2 \ldots A \text{ is located in cell } i, k \\ 0 & \text{Else} \end{cases}
- \( W_{ik}^{aaz} \) = Number of times a cell \( i, k \) is covered by static sonar type \( a \) with coverage strength \( a^z \).
- \( R_{ik} \) = \begin{cases} 1 & \text{If } 1 - \prod_{a^z}(1 - p_{a^z} * W_{ik}^{aaz}) \geq \gamma \\ 0 & \text{Else} \end{cases}

3.3 Constraints
- Budgetary Constraint: Available budget for static sonars
  \[ \sum_a c_a \sum_i \sum_k X_{ik}^a \leq \text{Available Budget} \quad \cdots (1) \]
- Cell coverage by sonars:
  For a cell \( (i,k) \) to be covered by detection strength \( a^z \), a sonar should exist to cover itself (Primary Coverage) and those in its neighborhood (Secondary and Tertiary Coverages):
  \[ \sum_a \sum_{(f,h) \in N_{ikaz}} X_{fh}^a + \sum_a X_{ik}^a \geq W_{ik}^{aaz} \quad \forall i, k, a^z \quad \cdots (2) \]
- Limitation in the total number of sonars covering a cell
  \[ \sum_a W_{ik}^{aaz} \leq \text{cover}_{\text{max}} \quad \forall i, j, a^z \quad \cdots (3) \]

Given that sonar detection probabilities are high (>=0.95), it is reasonable to limit the total number of sonars covering a cell to a small integer. An alternate approach is to limit the total detection probabilities attained by a cell \( i,k \) to a certain detection probability.
- Limitation in the total number of sonars allocated to a cell
  \[ \sum_a X_{ik}^a \leq 1 \quad \forall i, k \quad \cdots (4) \]

Limiting the number of sonar allotted to a cell \( i,k \) under budget limitation is reasonable and well documented in literature.
• Constraints to define $R_{ik}$

$$
(1 - \prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})) - \gamma < R_{ik} \quad \forall \ i, k, a
$$

$$
R_{ik} \leq (1 - \prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})) - \gamma + 1 \quad \forall \ i, k, a
$$

3.4 Objective Functions: Derivation and implied Constraints

Model #1: Maximize coverage solely based on Criticality factor

$$
Max \ast (1 - \prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})) \quad \ldots (5)
$$

$$
= Min \ast (\prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})) \quad \ldots (6)
$$

Equating equation (6) to an exponential function as shown in [41].

$$
e^{-Q_{ik}} = \prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})
$$

The objective equation becomes,

$$
Min \sum_{i} \sum_{k} h_{ik} \ast e^{-Q_{ik}} \quad \ldots (7)
$$

$$
== Max \sum_{i} \sum_{k} h_{ik} \ast Q_{ik} \quad \ldots (7')
$$

And the following constraints have to be added:

$$
e^{-Q_{ijk}} = \prod_{a^z}(1 - p_{a^z} \ast W_{ijk}^{a,a^z}) \quad \forall \ i, j, k, a \ldots (8)
$$

$$
Q_{ik} \geq 0
$$

However, constraint (8) can be linearized by taking its natural log:

$$
-Q_{ik} = \sum_{a^z} \ln(1 - p_{a^z})^{W_{ik}^{a,a^z}} \quad \forall \ i, k, a \ldots (8')
$$

$$
-Q_{ik} = \sum_{a^z} W_{ik}^{a,a^z} \ln(1 - p_{a^z}) \forall \ i, k, a \ldots (8'')
$$

Hence, model #1 can be modeled as a linear formulation (MIP).

Model #2: Maximize coverage (and the total number of covered regions attaining a detection probability threshold).

$$
Max \sum_{i} \sum_{k} h_{ik} \ast Q_{ik} \quad \ldots (7')
$$

Equations 9 and 10 enforces $R_{ik}^{a}$ to be either 0 or 1, depending on the value of detection probability in cell ik.

$$
(1 - \prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})) - \gamma < R_{ik} \quad \ldots (9) \quad \forall \ i, k
$$

$$
R_{ik} \leq (1 - \prod_{a^z}(1 - p_{a^z} \ast W_{ik}^{a,a^z})) - \gamma + 1 \quad \ldots (10) \quad \forall \ i, k
$$

$$
1 - e^{-Q_{ik}} - \gamma < R_{ik}^{a} \quad \forall \ i, k, a \ldots (9')
$$

$$
R_{ik} \leq (1 - e^{-Q_{ik}}) - \gamma + 1 \quad \forall \ i, k, a \ldots (10')
$$

Since equations 9’ and 10’ can’t be linearized by taking their natural logarithm, we model the problem as MINLP (Mixed Integer Non-linear Program).
In addition to all constraints applicable to model #1, model#2 includes constraints 9' and 10'; with objection function given as equation (7').

4. Small Scale Problem Implementation

4.1 Area of Interest
To implement the model, we consider a hypothetical but very realistic problem of vessels routes where sea routes are taken by different vessels. The area of interest is shown below:

![Hypothetic Marine area of interest](image)

<table>
<thead>
<tr>
<th>Key</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sea Route #1 (LNG Vessels)</td>
</tr>
<tr>
<td></td>
<td>Sea Route #2 (Oil Tankers)</td>
</tr>
<tr>
<td></td>
<td>Sea Route #3 (Steel)</td>
</tr>
<tr>
<td></td>
<td>Sea Route #4 (Agricultural produce)</td>
</tr>
<tr>
<td></td>
<td>Sea Route #5 (General Cargo)</td>
</tr>
<tr>
<td></td>
<td>Sea Route #6 (Vehicles)</td>
</tr>
</tbody>
</table>

**Figure 1** Hypothetic Marine area of interest

We discretize regions into grids and assign criticality indices to all the grids. The criticality is based on the type of sea route as well as the number of sea routes passing through a grid. For example, grids with vessels more vulnerable to attack such as LNG vessels are considered to be more important. As shown in the figure below, the numbers in each grid represent the criticality index using a hexagonal grid. Any well-defined utility function can be used in this regard.

4.2 Sonar Types and Coverage
For simplicity, the ranges of the sonars chosen are limited to coincide with their primary/secondary/tertiary coverage regions and these regions are taken to be unit grids.

Types of sonars considered (with different costs and coverage) include:
- Omni-directional (360° coverage) with coverage strength of 2 grid units (Primary and Secondary Detections).
• 180° Coverage with coverage strength of 3 grid units (Primary, Secondary and Tertiary Detections): Either Upper hemisphere or Lower hemisphere coverage.
• 90° Coverage with coverage strength of 3 grid units (Primary, Secondary and Tertiary Detections): 1st, 2nd, 3rd and 4th quadrant coverage.

Tables 1 and 2 below show major parameters used in the sample problem.

**Table 1**: Experiment Setting #1

<table>
<thead>
<tr>
<th>Sonar Type</th>
<th>Coverage Orientation</th>
<th>Coverage Range</th>
<th>Cost (thousands of $)</th>
<th>Detection Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type a1</td>
<td>360°</td>
<td>2 Grid Units</td>
<td>10.00</td>
<td>Primary: 0.99, Secondary: 0.78, Tertiary: N/A</td>
</tr>
<tr>
<td>Type a2</td>
<td>180°</td>
<td>3 Grid Units</td>
<td>10.00</td>
<td>Primary: 0.99, Secondary: 0.78, Tertiary: 0.58</td>
</tr>
<tr>
<td>Type a3</td>
<td>90°</td>
<td>3 Grid Units</td>
<td>10.00</td>
<td>Primary: 0.99, Secondary: 0.78, Tertiary: 0.58</td>
</tr>
</tbody>
</table>

Model #2

| Detection Probability Threshold, γ | 0.95 |

**Table 2**: Experiment Setting #2

<table>
<thead>
<tr>
<th>Sonar Type</th>
<th>Coverage Orientation</th>
<th>Coverage Range</th>
<th>Cost (thousands of $)</th>
<th>Detection Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type a1</td>
<td>360°</td>
<td>2 Grid Units</td>
<td>18.00</td>
<td>Primary: 0.99, Secondary: 0.78, Tertiary: N/A</td>
</tr>
<tr>
<td>Type a2</td>
<td>180°</td>
<td>3 Grid Units</td>
<td>15.00</td>
<td>Primary: 0.99, Secondary: 0.65, Tertiary: 0.45</td>
</tr>
<tr>
<td>Type a3</td>
<td>90°</td>
<td>3 Grid Units</td>
<td>12.00</td>
<td>Primary: 0.99, Secondary: 0.50, Tertiary: 0.35</td>
</tr>
</tbody>
</table>

Model #2

| Detection Probability Threshold, γ | 0.95 |

The implementation is done using GAMS/CPLEX.

4.3 **Objective Functions and Solver Solution Time**

Figures 2 & 3 and figures 4 & 5 respectively show the objective function and solver time for each model with the two experiment settings using two grid models. The unusual behavior of the solver solution time is due to the branching rules the CPLEX solver uses in its implementation. This phenomenon is well documented in literature. For example, [1] reports similar behavior.
Figure 2  Experiment Set #1: Objective Function and Solver Time (Model #1)

Figure 3  Experiment Set #2: Objective Function and Solver Time (Model #1)
4.4 Detection Coverage (Models #1 & #2)

In literature, different definitions exist for detection coverage as a performance measure. For our purpose, a grid/region is said to be covered if at least one sonar provides sonar coverage to it. Table 2 below shows percentage detection coverage achieved for both models with under some budgetary limits:

<table>
<thead>
<tr>
<th>Budget</th>
<th>Experimental Setting #1</th>
<th>Experimental Setting #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rectangular Grid</td>
<td>Hexagonal Grid</td>
</tr>
<tr>
<td>30</td>
<td>60%</td>
<td>68%</td>
</tr>
<tr>
<td>70</td>
<td>68%</td>
<td>100%</td>
</tr>
<tr>
<td>100</td>
<td>92%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Results Summary

- **Decision Maker Requirements of Detection Probabilities:**
  - Priority given to total area coverage: Model #1 outperforms model #2.
  - Priority given to regions/cells within area coverage: Model #2 outperforms model #1.

- **Run Time:**
  - Hexagonal Grids outperforms rectangular grids.
  - Model #1 outperforms model #2.

- **Objective Function**
  - Except for budget limit $30, Hexagonal grids outperforms rectangular grids. The reason for this exception is the extreme limited budget.
• **Sonar Coverage “Wastage”**
  (For our purpose, we define “coverage wastage” as sonar coverage accorded to regions outside our area of interest).
  • Hexagonal Grids outperforms Rectangular Grids.
  • Model#2 outperforms Model #1, giving coverage to regions outside the area of interest for some budgetary limits: *May be of little concern when solving a large scale problem.*

5 **Conclusion and Further Works**

In this proceedings paper, we provide an expanded review of papers related to sensor placement within the purview of risk analysis and the maritime domain. We present two different alternative models and show that each model becomes appropriate based on a specific decision maker requirement. We also implemented models to solve a hypothetical but practical problem in the maritime domain using two different grid approaches.

We are presently investigating the integration of sonar mobility and uncertainty into the models. In addition, we are working on the proposal of an appropriate heuristic to solve large scale problems and also provide an approximate linearization to model #2.

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**References**


